西北工業大學 NORTHWESTERN POLYTECHNICAL UNIVERSITY

ACCV 2022 Macau

Rolling Shutter Camera: Modeling, Optimization and Learning

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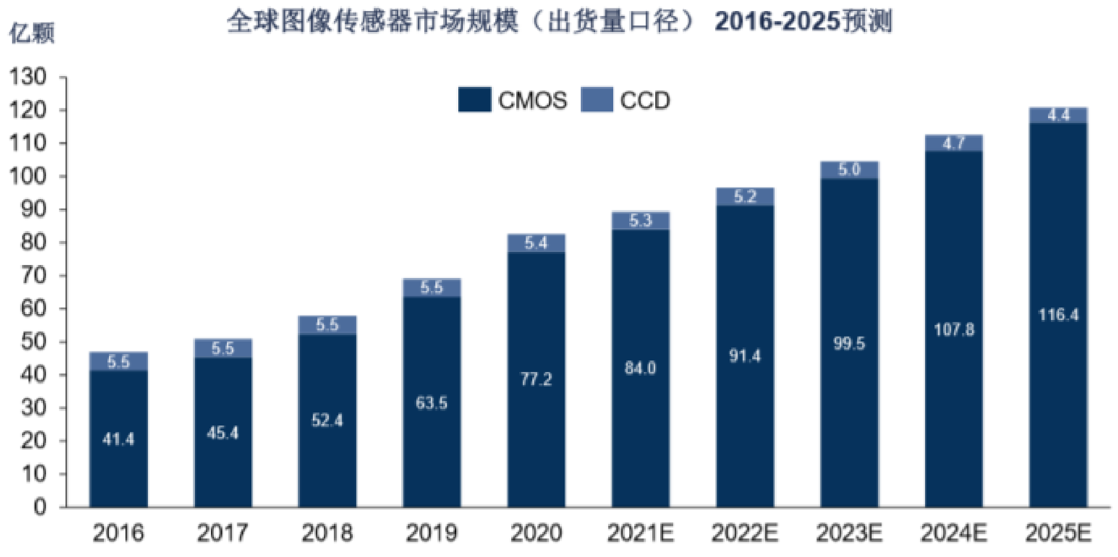
The Computer Vision and Robotics Group, CVR

Outline

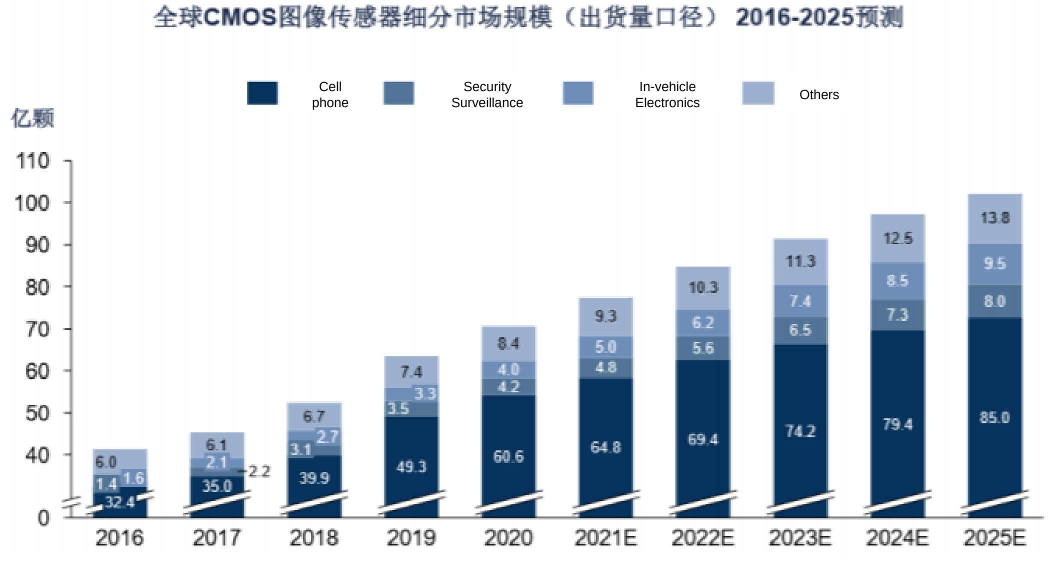
- **Introduction** (09:00-09:30)
- **Rolling Shutter Geometric Modeling and Optimization** (09:30-10:30)
  - Global Shutter Geometric Model
  - Rolling Shutter Uniform Motion Model
  - Rolling Shutter Differential Motion Model
  - Typical Applications
- **Learning-based Rolling Shutter Image Processing** (11:00-12:00)
  - Rolling Shutter Correction
  - Rolling Shutter Temporal Super-Resolution
  - Public Datasets
- **Futher Direction and Discussion** (12:00-13:00)

Introduction

CMOS vs. CCD:



Market size of CMOS and CCD



CMOS application classification

CMOS cameras where rolling shutter commonly used are now wining the battle of current camera market against to CCD cameras.

- ✓ Cheaper manufacturing (lower price)
- ✓ Allows on-chip processing
- ✓ Makes HD video affordable

《2019-2025全球与中国CMOS数码相机市场现状及未来发展趋势》

Introduction

Mobile phone

SLR camera

Kinect

Self-driving car

Camcorder

Robot platform

Drone

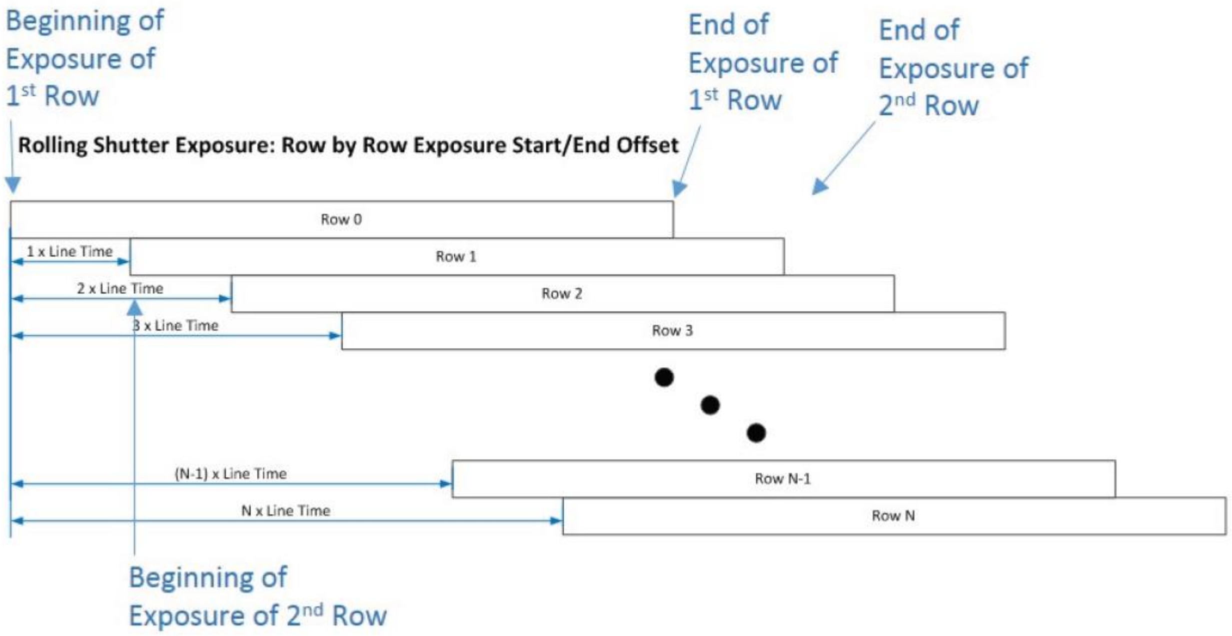
The first trend: "small pixel" technology:

- ✓ The popularity of H.265 encoding technology, the gradual uptake of 5 megapixel 4K products, and the rise of intelligent video needs such as face recognition and object recognition.
- ✓ The number of pixel dots is increasing, the pixel size is shrinking, and the clarity continues to improve.
- ✓ Future market demand for CMOS image sensors to support **higher resolution and higher frame rate output** is increasingly urgent.

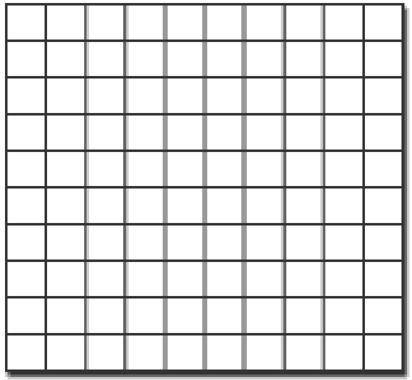
Introduction

Rolling shutter mechanism

Unlike a global shutter camera capturing all pixels simultaneously using a CCD sensor, pixels on the rolling shutter CMOS sensor plane are exposed commonly **from top to bottom in a row-by-row fashion** with a constant inter-row delay.

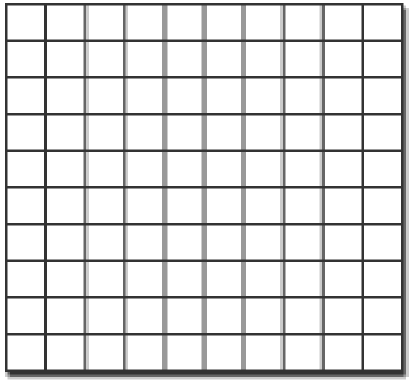


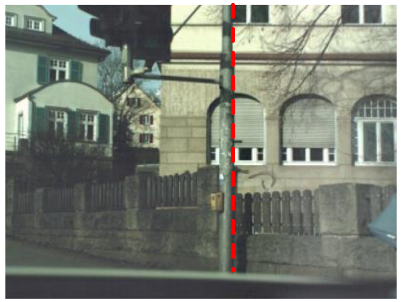
Rolling Shutter



Rolling shutter image

Global Shutter



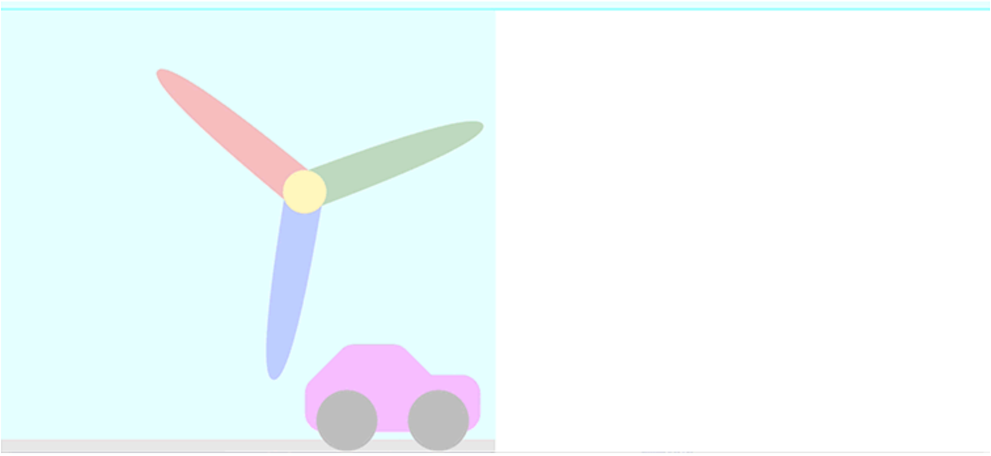


Global shutter image

Introduction

Rolling shutter effect

- Create some unintended geometric distortions if you're filming fast-moving subjects or panning your video camera across a scene, such as skew, wobble, etc.
- Common in footage from DSLRs and mobile phone cameras.



When the rolling shutter effect relevant for computer vision:

- 3D modeling from images;
- Visual SLAM;
- Video stabilization algorithms, Video panoramas, etc.;
- Any geometric measurement from images.

Introduction

Sony A7M4

- Release date: 2021.10
- Full-frame back-illuminated CMOS

4K, 25fps, *Severe distortion*

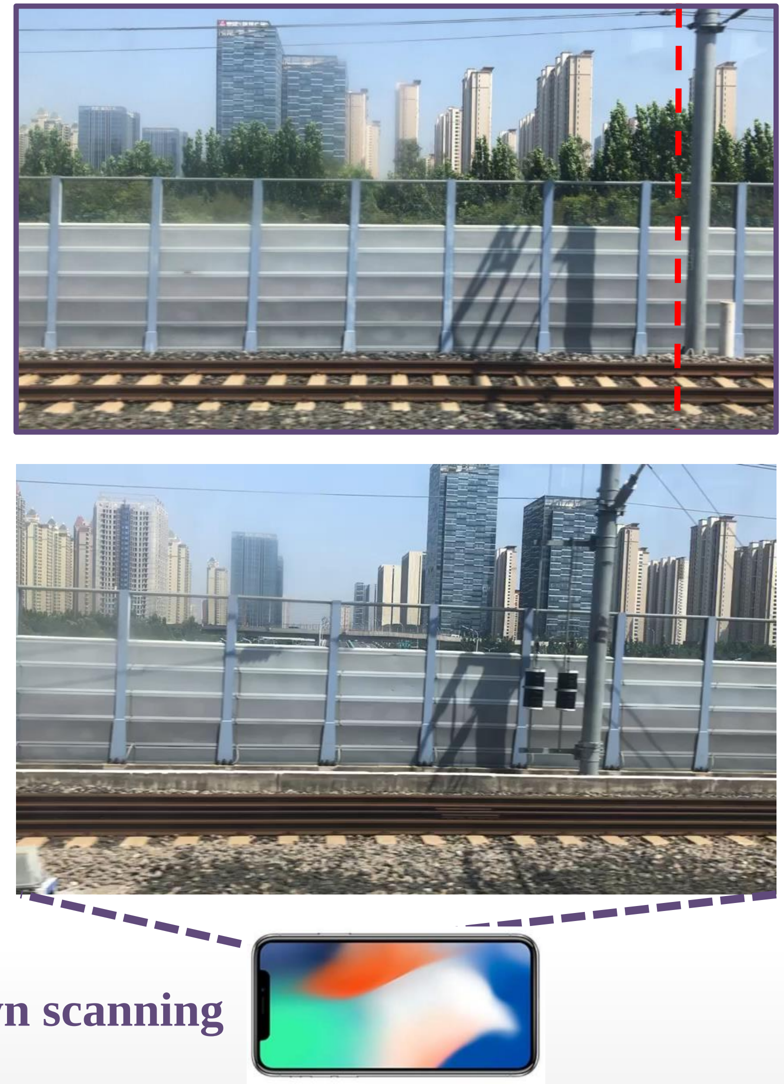
1080p, 25fps, Moderate distortion

DJI Ronin 4D

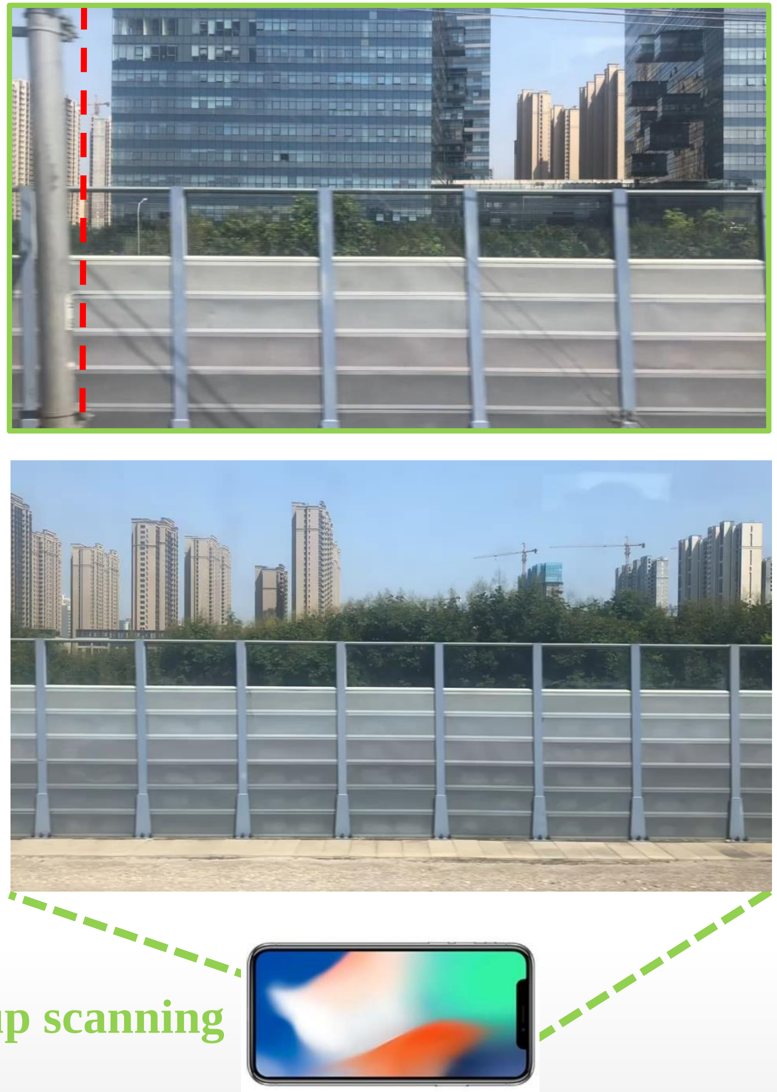
- Release date: 2021.10
- Electronic Rolling Shutter

Introduction

◆ Iphone X



Top-down scanning



Bottom-up scanning

Introduction

Importance of rolling shutter effect removal

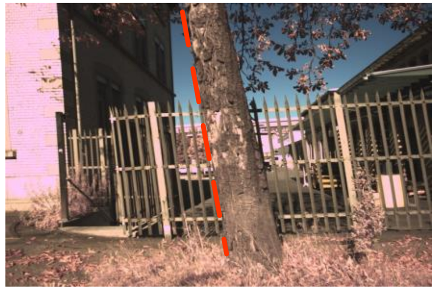
Original rolling shutter input

Corrected global shutter input

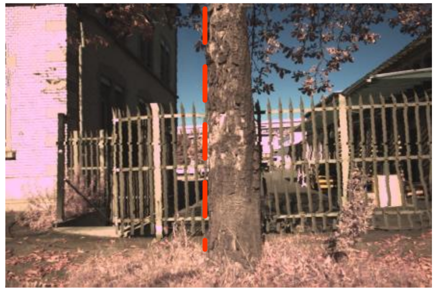
Zhuang B, Cheong L F, Hee Lee G. Rolling-shutter-aware differential sfm and image rectification. ICCV 2017.

Introduction

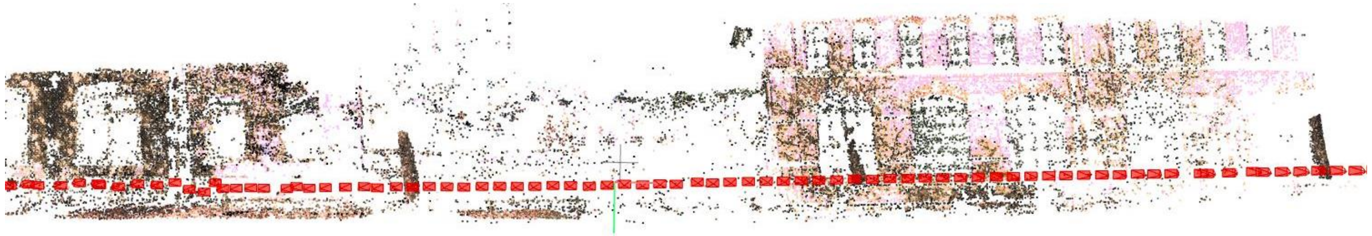
Importance of rolling shutter effect removal



Rolling shutter image



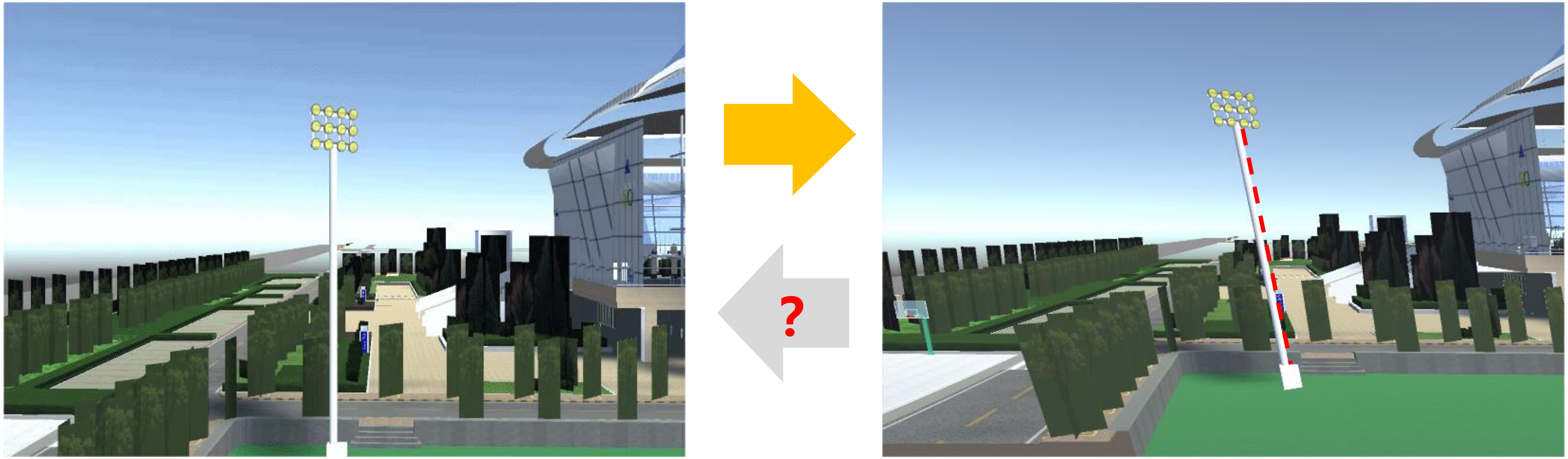
Global shutter image



3D reconstruction result

Albl C, Kukelova Z, Larsson V, et al. From two rolling shutters to one global shutter. CVPR 2020.

Importance of rolling shutter effect removal



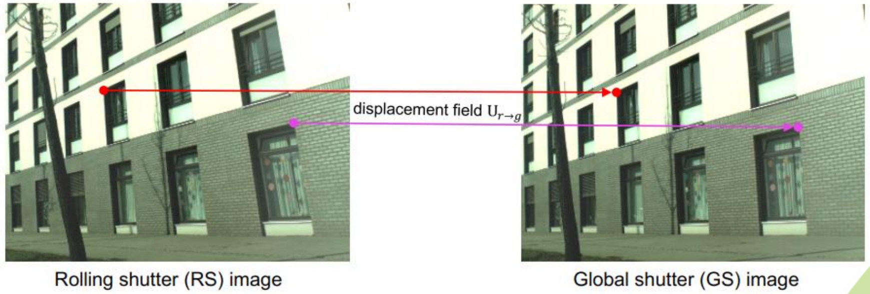
Latent global shutter image sequence

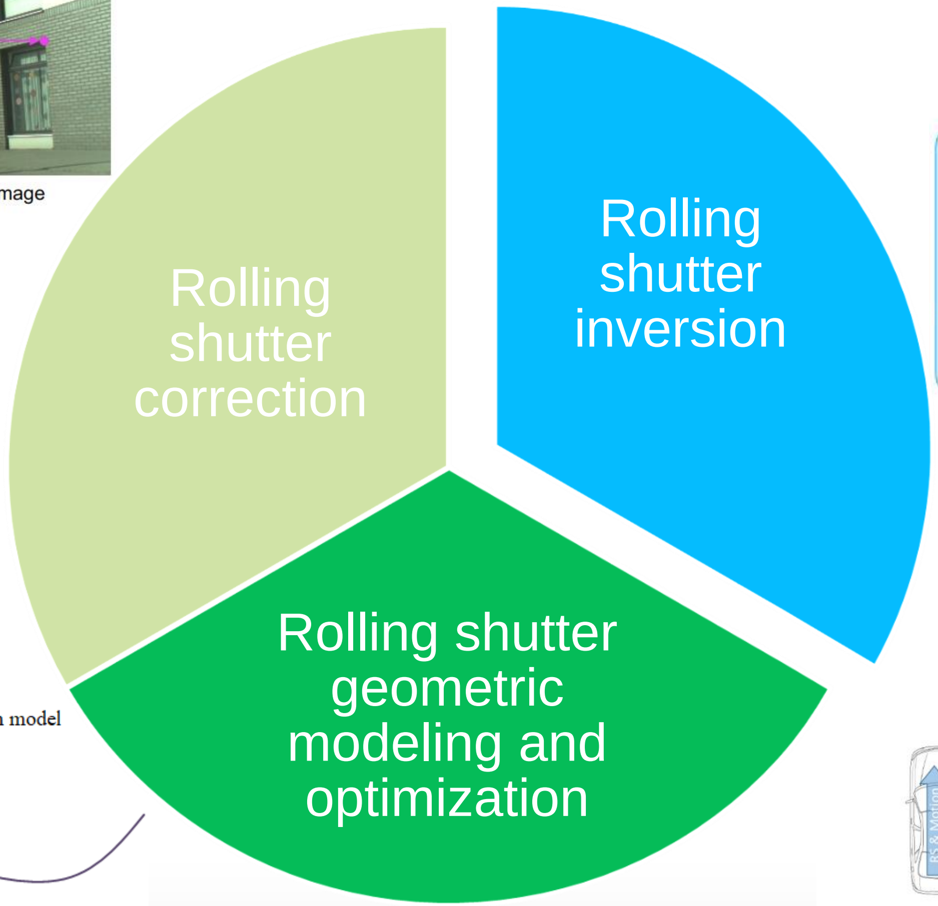
Rolling shutter image

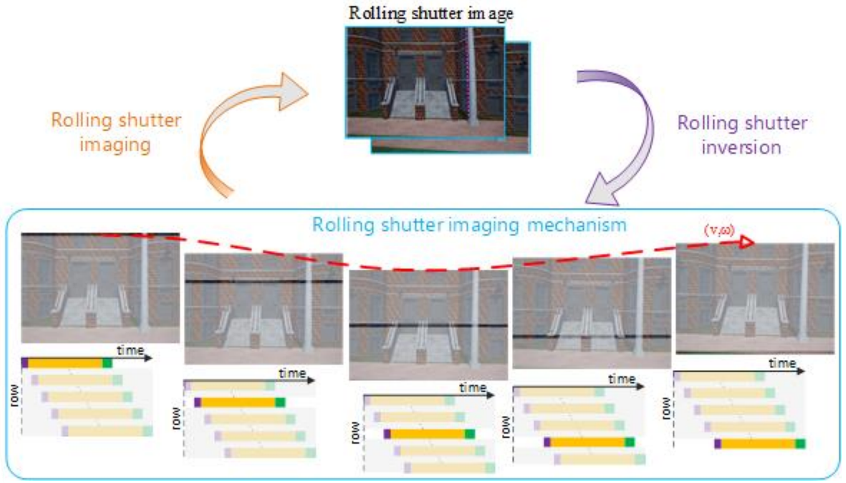
- ✓ The rolling shutter images implicitly contain **rich high framerate temporal dynamic observation information**, i.e., camera motion information (temporally) and scene 3D information (spatially).
- ✓ It is beneficial to **achieve high framerate video reconstruction and high quality 3D reconstruction** in the framework of temporal dynamic modeling and deep learning.
- ✓ This is of great importance for practical applications such as computational photography, visual tracking, scene understanding, image entertainment, novel view synthesis, video editing and compression.

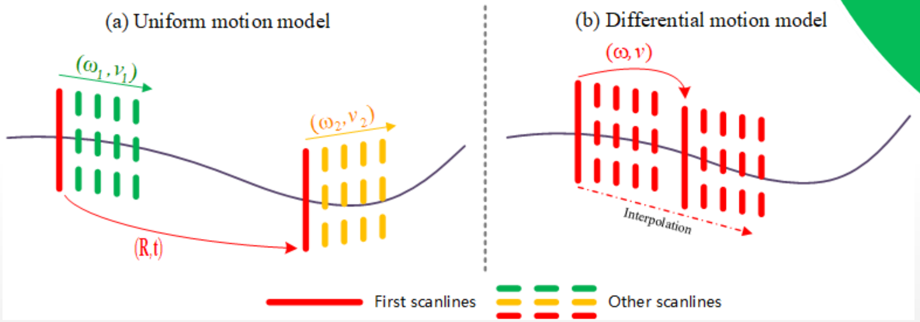
Introduction

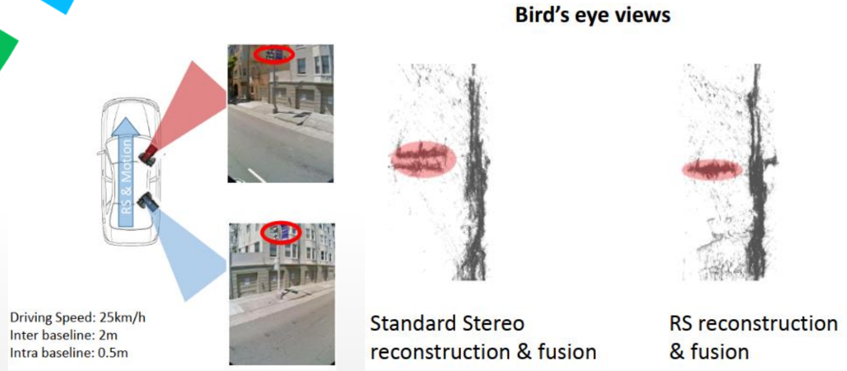
Overview: Rolling shutter geometric problem and image processing







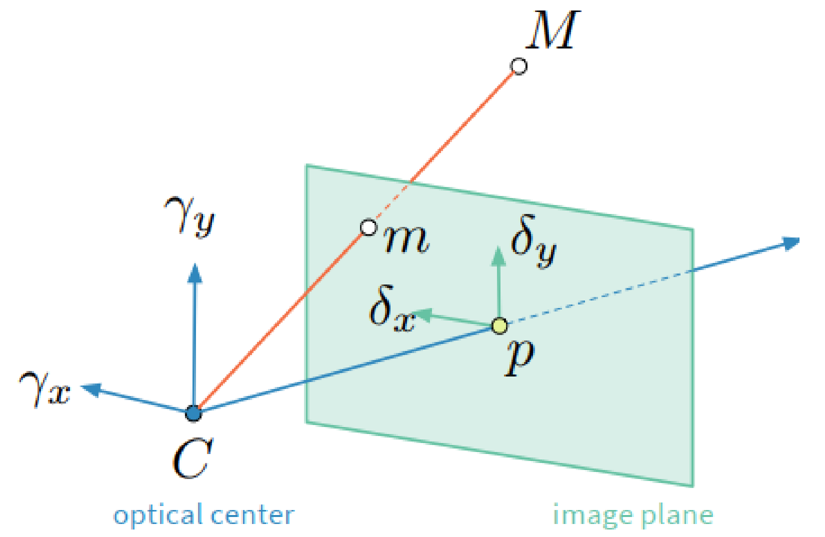


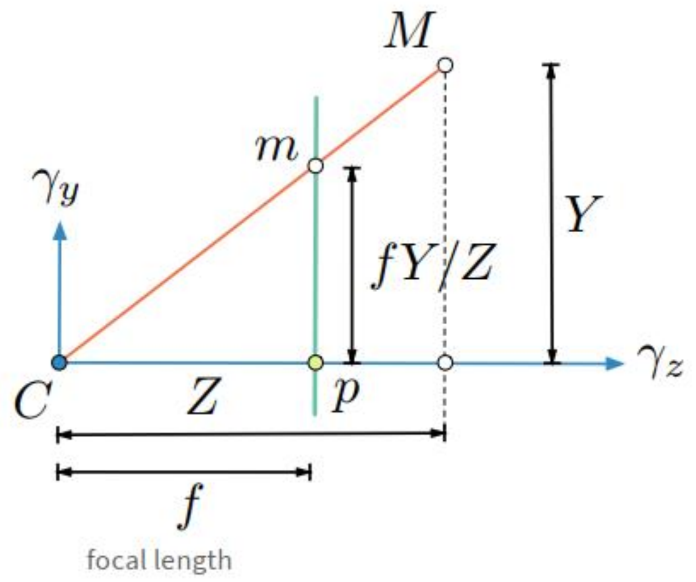


Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: Pinhole camera geometry

- Is described by its optical center C and the image plane ϕ.
- The distance of the image plane from C is the f, the focal length.
- The relation between M the 3D coordinates of a scene point and m the coordinates of its projection onto the image plane is described by the perspective projection.





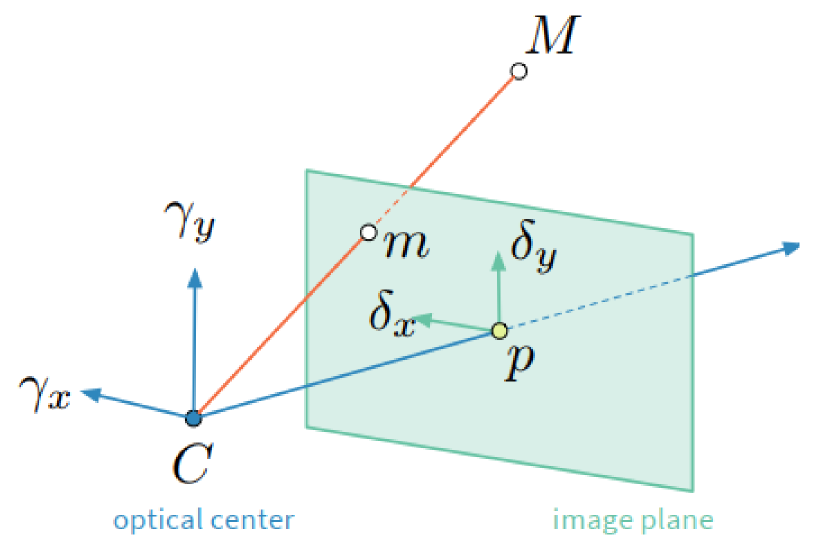
Luca Magri, Federica Arrigoni. Inside Plato's door: a tour in Multi-view Geometry. Tutorial at CVPR 2022.

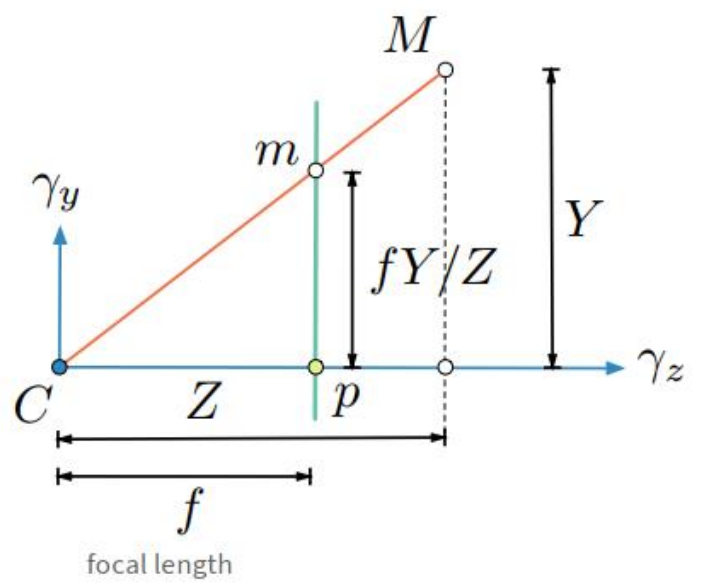
Rolling Shutter Geometric Modeling and Optimization

- Global shutter geometric model: Pinhole camera geometry

Fix a Cartesian coordinate system $\{\gamma_x, \gamma_y, \gamma_z\}$ in the optical center, with γ_z perpendicular to the image plane. By similar triangles, $M = (X_M, Y_M, Z_M)$ is mapped to point $m = (\frac{fX_M}{Z_M}, \frac{fY_M}{Z_M})$

$$\boldsymbol{M} = (X_M, Y_M, Z_M) \mapsto \boldsymbol{m} = (x_m, y_m), \text{where} \begin{cases} x_m = f\, X_M/Z_M \\ y_m = f\, Y_M/Z_M \end{cases}$$

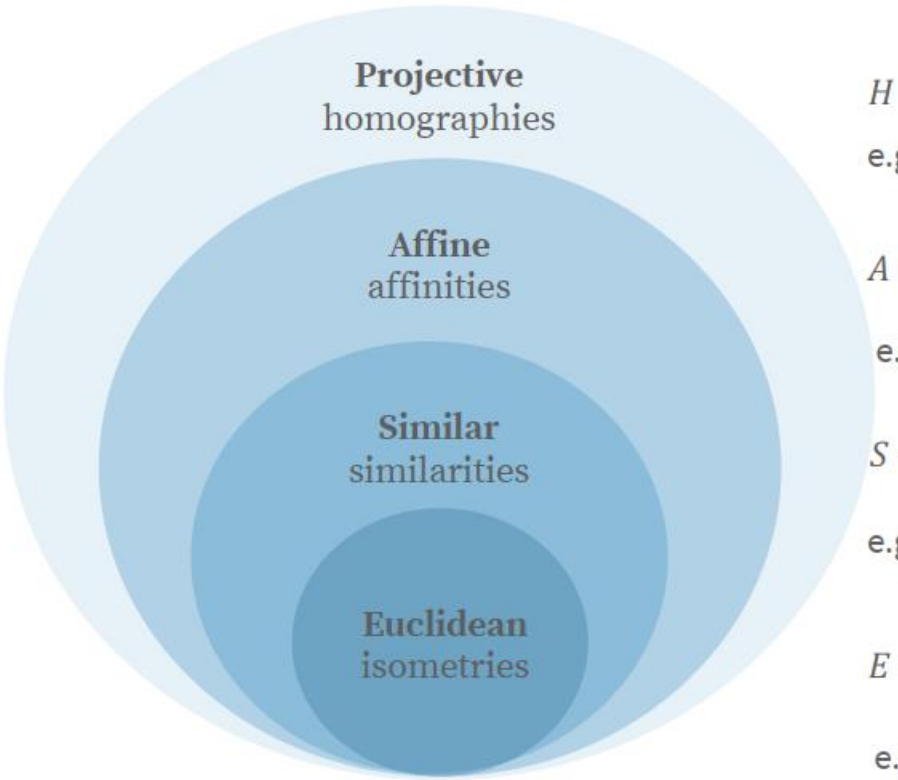




Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: A hierarchy of transformations

- According to Erlangen Program, due to Felix Klein (1872) geometry is the study of properties that are invariant with respect to a certain group of transformations.



$H \in GL(3) = \{H \in \mathbb{R}^{3\times3} \ \det(H) \neq 0\}$

e.g., incidence, collineations, tangency

$A = \begin{bmatrix} B & \boldsymbol{c} \\ \boldsymbol{0} & 1 \end{bmatrix} \quad B \in \mathbb{R}^{2\times2}$

e.g., ratio of length of parallel lines, parallelism, ratio of area

$S = \begin{bmatrix} sR & \boldsymbol{t} \\ \boldsymbol{0} & 1 \end{bmatrix} \quad R \in \mathbb{R}^{2\times2} \ R^TR = I$

e.g., angle, ratio of lengths, ratio of areas

$E = \begin{bmatrix} R & \boldsymbol{t} \\ \boldsymbol{0} & 1 \end{bmatrix} \in \mathrm{SE}(2) \quad R \in \mathbb{R}^{2\times2} \ R^TR = I$

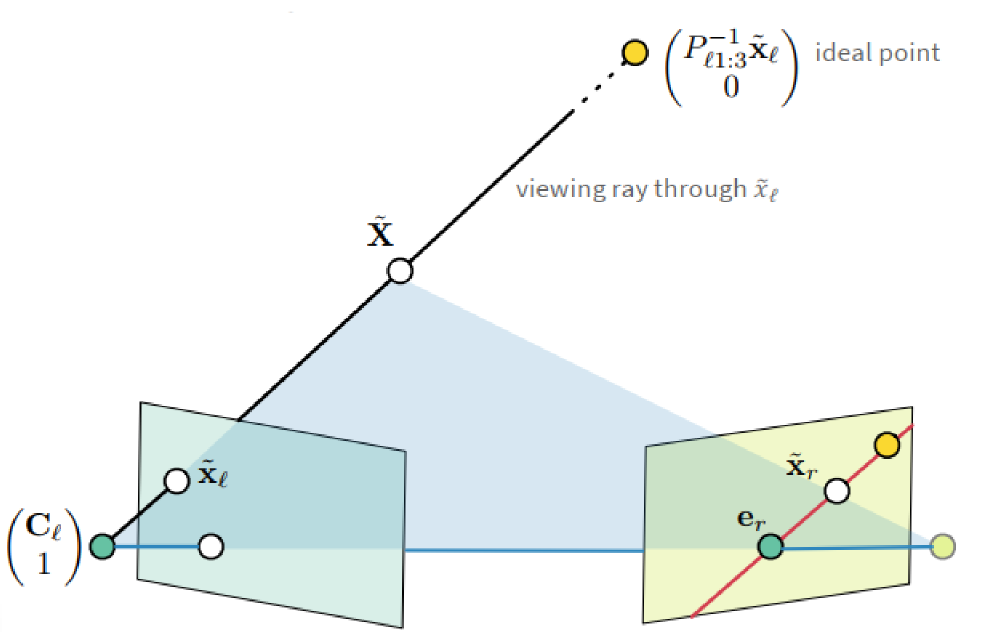
e.g., length, angle, area

Luca Magri, Federica Arrigoni. Inside Plato's door: a tour in Multi-view Geometry. Tutorial at CVPR 2022.

Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: Fundamental matrix & Essential matrix

Fundametal matrix:
the fundamental matrix F is the unique 3×3 matrix rank 2 homogeneous matrix which satisfy $\boldsymbol{x}_r^T F \boldsymbol{x}_l = 0$ for all corresponding points $\boldsymbol{x}_r \leftrightarrow \boldsymbol{x}_l$ in the two images



- The fundamental matrix can be thought as the generalization of the essential matrix in which the (inessential) assumptions on camera calibration have been removed.

Given a pair of cameras, two relations hold:

$$\boldsymbol{x}_r^T F \boldsymbol{x}_\ell = 0 \text{ and } \boldsymbol{p}_r^T E \boldsymbol{p}_\ell = 0$$

where $\boldsymbol{p} = K^{-1}\boldsymbol{x}$.

Combining these we get

$$\boldsymbol{x}_r^T K_r^{-1} E K_\ell \, \boldsymbol{x}_\ell = 0$$

which implies

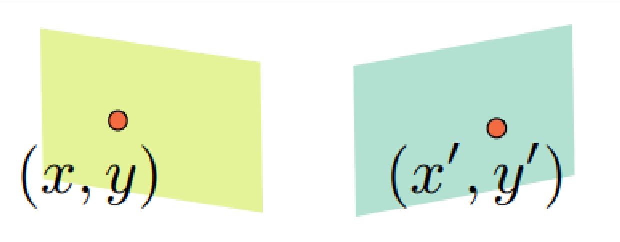
$$E = K_r^T F K_\ell$$

Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: Fundamental matrix & Essential matrix

Two-view correspondences and epipolar line

Geometric interpretation



$$\begin{pmatrix} x & y & x' & y' & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & f_{11} & f_{21} & f_{31} \\ 0 & 0 & f_{12} & f_{22} & f_{32} \\ f_{11} & f_{12} & 0 & 0 & f_{13} \\ f_{21} & f_{22} & 0 & 0 & f_{23} \\ f_{31} & f_{32} & f_{13} & f_{23} & 2f_{33} \end{bmatrix} \begin{pmatrix} x \\ y \\ x' \\ y' \\ 1 \end{pmatrix} = 0$$

Uncalibrated view: the fundamental matrix

- 8 points algorithm
- 7 points algorithm

Calibrated view: the essential matrix

- 8 points algorithm
- 5 points algorithm (idea)

Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: The eight-point algorithm

Given a set of correspondences $\{\boldsymbol{x}_{i\ell} \leftrightarrow \boldsymbol{x}_{ir}\}$, we want to determine the matrix F that encodes the bilinear condition: $\boldsymbol{x}_{ir}^T F \boldsymbol{x}_{i\ell} = 0$

This matrix can be recovered using the property of the Kronecker product:

$$\boldsymbol{x}_{ir}^T F \boldsymbol{x}_{i\ell} = 0 \Leftrightarrow \operatorname{vec}\left(\boldsymbol{x}_{ir}^T F \boldsymbol{x}_{i\ell}\right) = 0 \Leftrightarrow \left(\boldsymbol{x}_{i\ell}^T \otimes \boldsymbol{x}_{ir}^T\right)\operatorname{vec}(F) = 0$$

Every correspondence yields a homogeneous equation in the 9 unknown of F. From n corresponding points we get the system:

$$\underbrace{\begin{bmatrix} \boldsymbol{x}_{1\ell}^T \otimes \boldsymbol{x}_{1r}^T \\ \boldsymbol{x}_{2\ell}^T \otimes \boldsymbol{x}_{2r}^T \\ \vdots \\ \boldsymbol{x}_{n\ell}^T \otimes \boldsymbol{x}_{nr}^T \end{bmatrix}}_{A_n} \operatorname{vec}(F) = 0.$$

The solution of this system is the $\ker(A_n)$. When the points are in general position and $n = 8$, the solution is determined up to a multiplicative factor. In practice, when more than 8 points are available the solution can be obtained using the SVD.

Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: The eight-point algorithm

The matrix F estimated from the system $A_8 \text{vec}(F) = 0$, in general does not have $\text{rank}(F) = 2$.

The rank-2 condition can be enforced using the following

> Theorem (Eckart-Young). Let A be a $m \times n$ matrix of rank r and be $A = U_r D V_r^T$ be its compact singular value decomposition: $A = \sum_{i=1}^{r} \sigma_i u_i v_i^T$.
> The rank-k matrix closest to A in Frobenius norm is the matrix $A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$.

thus computing the SVD of the estimated F and considering the closest rank-2 matrix in Frobenius norm.

Alternatively, the rank-2 condition can be enforced directly by construction using the seven-points algorithm.

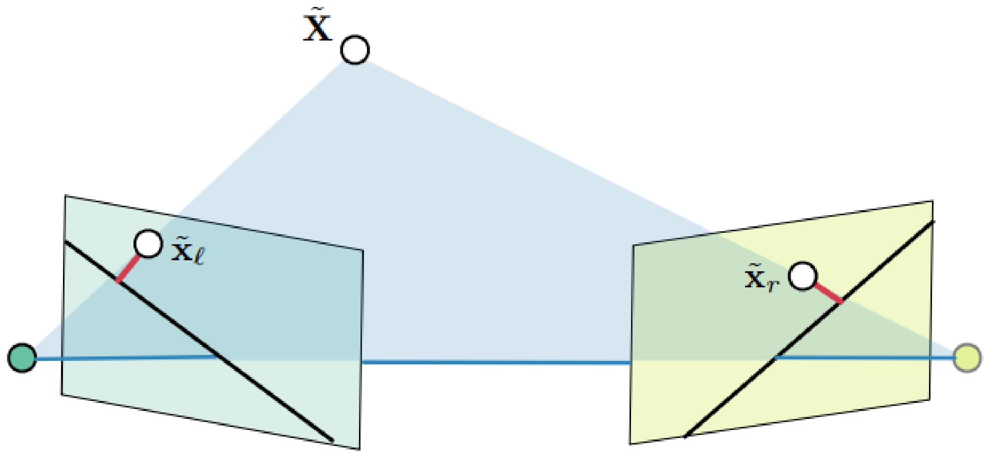
Rolling Shutter Geometric Modeling and Optimization

Global shutter geometric model: Nonlinear refinement

- Instead of minimizing an algebraic error, it is better to minimize geometric errors that can be expressed in terms of the distances between points and their corresponding epipolar lines:

$$\sum d\left(F\boldsymbol{x}_\ell, \boldsymbol{x}_r\right)^2 + d\left(F^T\boldsymbol{x}_r, \boldsymbol{x}_\ell\right)^2$$

where d is the Euclidean distance between a point and a line.



Residuals for the fundamental matrix

- ✓ Algebraic distance: $\boldsymbol{x}_r^\top F\boldsymbol{x}_\ell$
- ✓ Symmetric epipolar distance: $\sum d\left(F\boldsymbol{x}_\ell, \boldsymbol{x}_r\right)^2 + d\left(F^T\boldsymbol{x}_r, \boldsymbol{x}_\ell\right)^2$
- ✓ Sampson distance (the geometric distance to the first order approximation of the curve):

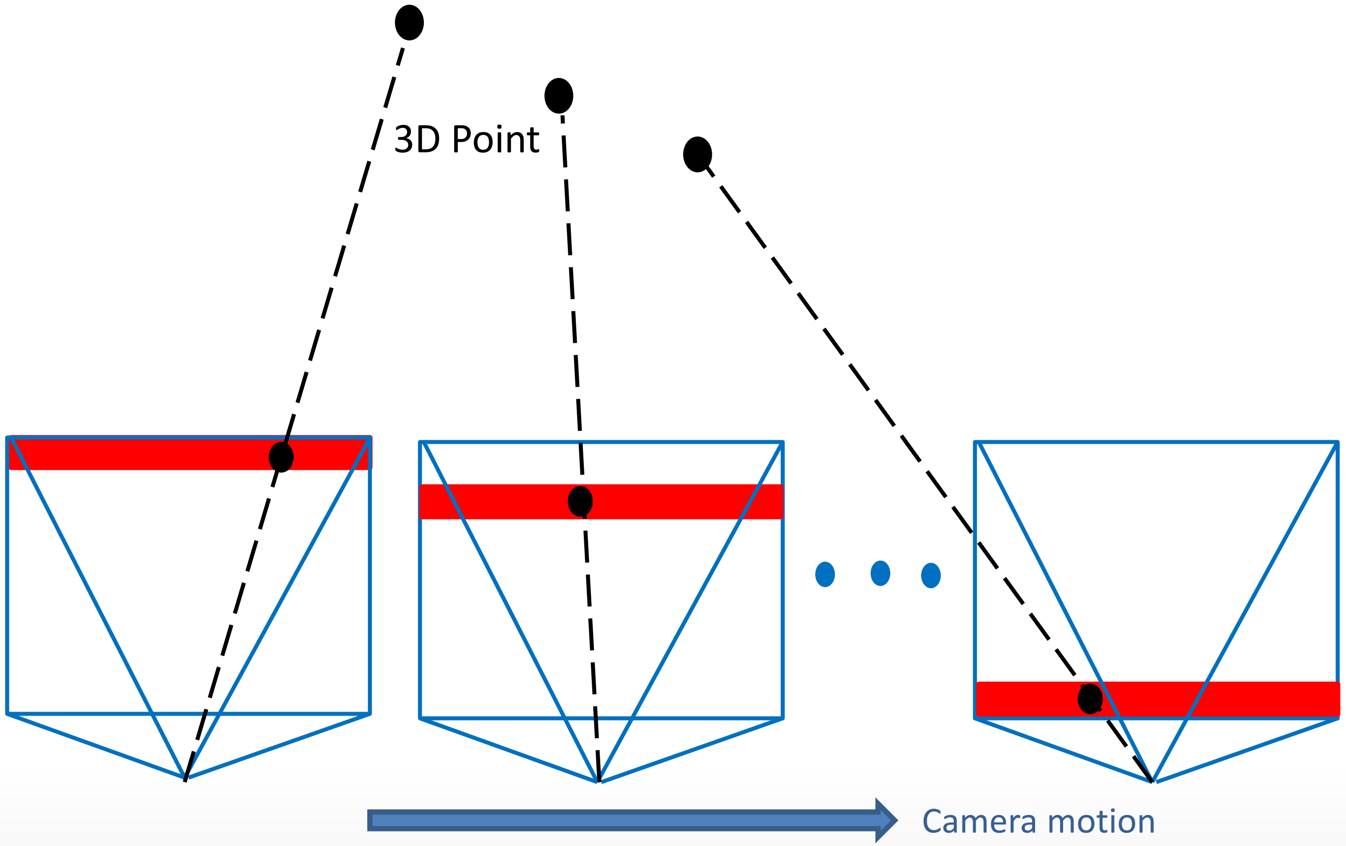
$$\frac{(\boldsymbol{x}_r^\top F\boldsymbol{x}_\ell)^2}{(F\boldsymbol{x}_\ell)_1^2 + (F\boldsymbol{x}_\ell)_2^2 + (F^\top\boldsymbol{x}_r)_1^2 + (F^\top\boldsymbol{x}_r)_2^2}$$

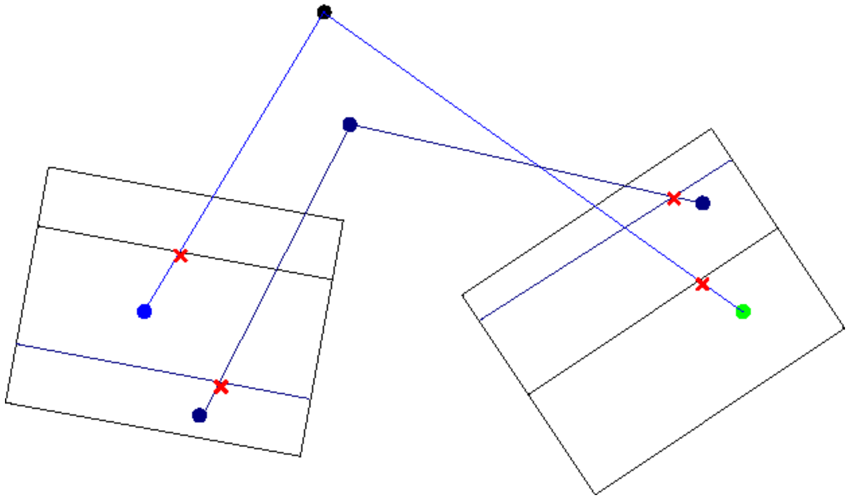
- ✓ Reprojection error (distance to the “epipolar cone”): $\min_{p,q} d(x_\ell, p)^2 + d(x_r, q)^2 \text{ subject to } q^\top F p = 0$

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter geometric modeling

- Due to the temporal-dynamic exposure characteristics of the rolling shutter camera, each of its scanlines usually possesses a different projection center, i.e., a series of latent local frames.





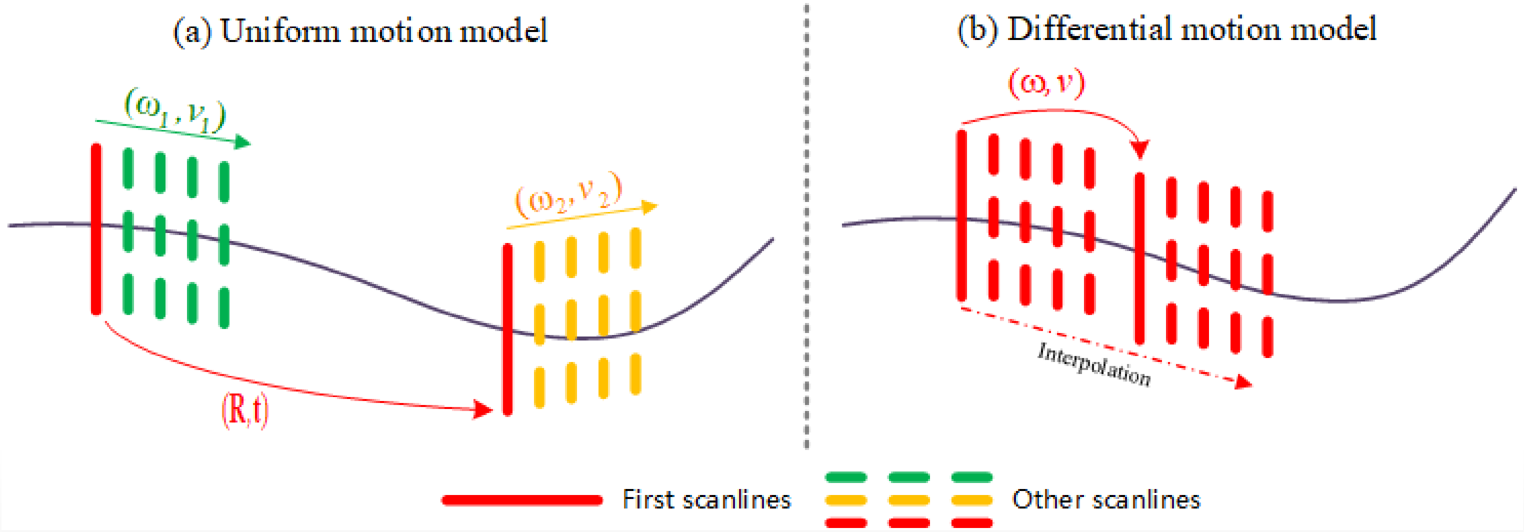
Different scanlines have different projection centers. For any pair of correspondences (indicated by 'x'), the co-planarity constraint still holds.

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter geometric modeling

Suppose that the local poses of each scanline of a general rolling shutter camera trace out a smooth trajectory in the SE(3) space.

- (a) Uniform motion model is mainly used for various minimal solver problems (e.g., relative/absolute pose estimation), combined with the discrete epipolar geometry method and the discrete 3D-2D projection method.
- (b) Differential motion model is more suitable for adjacent frame motion modeling, combined with the differential epipolar geometry method and the differential 3D-2D projection method.



Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Uniform motion model

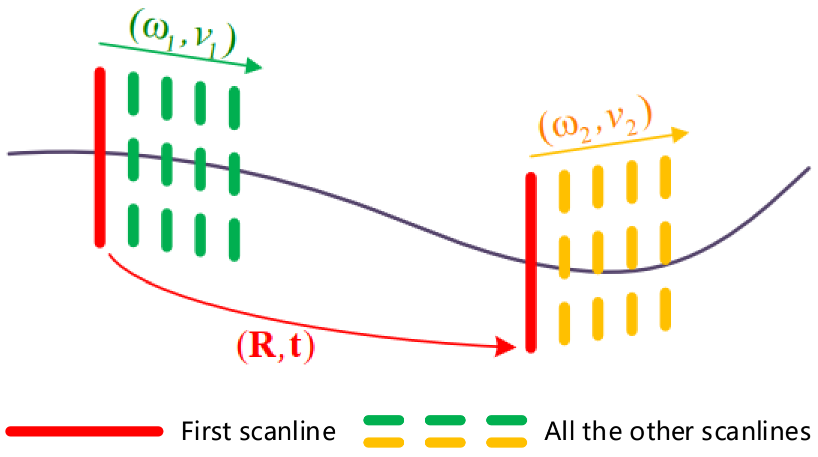
Modeling

Assume that the smoothly moving camera rotates at a constant angular velocity $\boldsymbol{\omega} \in \mathbb{R}^3$ and translates at a constant linear velocity $\boldsymbol{v} \in \mathbb{R}^3$ at the same time.

Assume that the first scanline of the rolling shutter image has 6 DoF absolute poses $\mathbf{R}_0 \in \mathrm{SO}(3)$ and $\boldsymbol{t}_0 \in \mathbb{R}^3$ in the world coordinate system

The absolute camera poses $\mathbf{P}_s = [\mathbf{R}_s, \mathbf{t}_s]$ of s-th scanline satisfy:

$$\mathbf{R}_s = (\mathbf{I} + \sin(s\omega)\,[\boldsymbol{n}]_\times + (1-\cos(s\omega))\,[\boldsymbol{n}]_\times^2)\mathbf{R}_0,$$
$$\mathbf{t}_s = \mathbf{t}_0 + s\boldsymbol{v},$$



Meingast M, Geyer C, Sastry S. Geometric models of rolling-shutter cameras. arXiv preprint cs/0503076, 2005.

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Uniform motion model

Modeling

Since the camera typically has a rapid scanning time, it is reasonable to make the assumption that the inter-scanline rotation displacement is sufficiently small.

Using the small-rotation approximation yields **the uniform motion model**:

$$\mathbf{R}_s = (\mathbf{I} + s[\boldsymbol{\omega}]_\times)\mathbf{R}_0,$$
$$\mathbf{t}_s = \mathbf{t}_0 + s\boldsymbol{v}.$$

All the projection centers will form a spiral 3D trajectory.

Uniform motion model and its variants

Motion	Pose $\mathbf{P}_s$	Application Examples
linear	$[\mathbf{I},\ s\boldsymbol{v}]$	*e.g.* vehicles traveling in a straight line
orbital	$[\mathbf{I} + s[\boldsymbol{\omega}]_\times,\ \boldsymbol{v}]$	*e.g.* video clip taken by hand-held devices
spiral	$[\mathbf{I} + s[\boldsymbol{\omega}]_\times,\ s\boldsymbol{v}]$	*e.g.* general RS cameras with smooth motion
linear	$[\mathbf{I} + s[\boldsymbol{\omega}]_\times,\ -s(\mathbf{I} + s[\boldsymbol{\omega}]_\times)\boldsymbol{v}]$	*e.g.* 3D-2D projection geometry based on continuous video sequences

Saurer O, Koser K, Bouguet J Y, et al. Rolling shutter stereo. CVPR 2013.

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Uniform motion model

Optimization

Given N pairs of 3D-2D correspondences, including the 3D point coordinate $\mathbf{X}_i \in \mathbb{R}^3$ (in the world coordinate system) as well as the corresponding 2D image coordinate $\mathbf{x}_i = (u_i, v_i) \in \mathbb{R}^2$, we can obtain the absolute pose of $\mathbf{x}_i$ as $\mathbf{P}_{u_i} = [\mathbf{R}_{u_i}, \mathbf{t}_{u_i}]$.

Consequently, the RS-aware re-projection error can be derived as

$$\boldsymbol{v}^*, \boldsymbol{\omega}^* = \underset{\boldsymbol{v}, \boldsymbol{\omega}}{\arg\min} \sum_{i=1}^{N} \|\mathbf{x}_i - \pi(\mathbf{X}_i, \mathbf{P}_{u_i})\|_2^2, \tag{1}$$

where $\pi(\cdot) : \mathbb{P}^3 \rightarrow \mathbb{P}^2$ denotes the projection function, defined as

$$\begin{aligned} \pi(\mathbf{X}_i, \mathbf{P}_{u_i}) &= \langle \mathbf{K}(\mathbf{R}_{u_i}\mathbf{X}_i + \mathbf{t}_{u_i}) \rangle, \\ \langle (x,\ y,\ z)^\top \rangle &= (x/z,\ y/z)^\top. \end{aligned} \tag{2}$$

Here, $\mathbf{K}$ is the intrinsic matrix whose calibration is easy to implement, e.g., by applying any standard camera calibration procedure to a still scene image captured by a stationary RS camera.

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Differential motion model

Modeling (Motion parameterization)

Assume that there is a relatively small inter-frame camera velocity $(\boldsymbol{v}, \boldsymbol{\omega})$ between the first two scanlines of two consecutive RS frames. Then, the intra-frame camera motions of all other scanlines can be obtained by interpolation.

Formally, the absolute camera position and rotation $(\mathbf{p}_1^{s_1}, \mathbf{r}_1^{s_1})$ (resp. $(\mathbf{p}_2^{s_2}, \mathbf{r}_2^{s_2})$) of the s_1-th (resp. s_2-th) scanline in frame 1 (resp. frame 2) *w.r.t.* the first scanline of frame 1 can be expressed as:

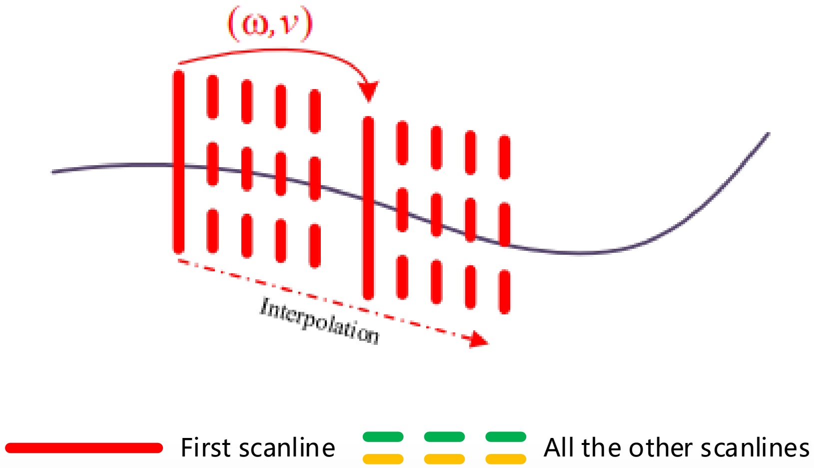
$$\mathbf{p}_1^{s_1} = \lambda_1^{s_1}\boldsymbol{v}, \quad \mathbf{r}_1^{s_1} = \lambda_1^{s_1}\boldsymbol{\omega},$$
$$\mathbf{p}_2^{s_2} = \lambda_2^{s_2}\boldsymbol{v}, \quad \mathbf{r}_2^{s_2} = \lambda_2^{s_2}\boldsymbol{\omega},$$

where $\lambda_1^{s_1}$ and $\lambda_2^{s_2}$ denote the **interpolation factors**.

Therefore, the relative motion between the s_1-th and s_2-th scanlines will satisfy

$$\boldsymbol{v}_{s_1 s_2} = \mathbf{p}_2^{s_2} - \mathbf{p}_1^{s_1} = (\lambda_2^{s_2} - \lambda_1^{s_1})\,\boldsymbol{v},$$

$$\boldsymbol{\omega}_{s_1 s_2} = \mathbf{r}_2^{s_2} - \mathbf{r}_1^{s_1} = (\lambda_2^{s_2} - \lambda_1^{s_1})\,\boldsymbol{\omega}.$$



Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Differential motion model

Modeling (Linear interpolation)

To efficiently model the above interpolation factor, Zhuang *et al.* proposed a **linear interpolation** under the assumption of constant velocity motion, *i.e.*

$$\begin{aligned} \lambda_1^{s_1} &= \frac{\gamma s_1}{h}, \\ \lambda_2^{s_2} &= 1 + \frac{\gamma s_2}{h}. \end{aligned} \tag{3}$$

Here γ is the readout time ratio, which indicates the ratio between the total readout time and the total frame time (including inter-frame idle time). h is the total scanline number in an RS image.

Since $s_2 - s_1$ can be expressed by the vertical optical flow $\mathbf{f}_v$, *i.e.*

$$\lambda_2^{s_2} - \lambda_1^{s_1} = 1 + \frac{\gamma \mathbf{f}_v}{h}, \tag{4}$$

the scanline-varying camera poses can be recovered through a simple linear scaling operation.

Zhuang B, Cheong L F, Hee Lee G. Rolling-shutter-aware differential sfm and image rectification. ICCV 2017.

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Differential motion model

Modeling (Quadratic interpolation)

Further and more generally, under the constant acceleration motion assumption, a **quadratic interpolation** was also proposed by Zhuang *et al.* , *i.e.*

$$\begin{aligned} \lambda_1^{s_1} &= \frac{2}{k+2}\left(\frac{\gamma s_1}{h} + \frac{k}{2}\left(\frac{\gamma s_1}{h}\right)^2\right), \\ \lambda_2^{s_2} &= \frac{2}{k+2}\left(1 + \frac{\gamma s_2}{h} + \frac{k}{2}\left(1 + \frac{\gamma s_2}{h}\right)^2\right), \end{aligned} \quad (5)$$

where k denotes the acceleration factor and is in the same direction as the camera velocity, *i.e.*, $k > 0$ for acceleration and $k < 0$ for deceleration. Note that k needs to be estimated additionally when used.

Linear interpolation is a special case of quadratic interpolation when k=0.

Zhuang B, Cheong L F, Hee Lee G. Rolling-shutter-aware differential sfm and image rectification. ICCV 2017.

Rolling Shutter Geometric Modeling and Optimization

Rolling shutter model: Differential motion model

Optimization

The RS-aware differential re-projection error can be developed as

$$\boldsymbol{v}^*, \boldsymbol{\omega}^* = \arg\min_{\boldsymbol{v},\boldsymbol{\omega}} \sum_{i=1}^{N} \left\| \mathbf{f}_i - \beta_i \left(\frac{\mathbf{A}_i \boldsymbol{v}}{Z_i} + \mathbf{B}_i \boldsymbol{\omega} \right) \right\|_2^2, \tag{6}$$

where $\beta_i = \lambda_2^{s_2^i} - \lambda_1^{s_1^i}$, and the normalized image point $\mathbf{x}_i$ in scanline s_1^i corresponds to a forward optical flow of $\mathbf{f}^i = (\mathbf{f}_u^i, \mathbf{f}_v^i)$ and corresponds to a 3D point of depth Z_i,

$$\mathbf{A}_i = \begin{bmatrix} -f & 0 & x_i \\ 0 & -f & y_i \end{bmatrix},$$

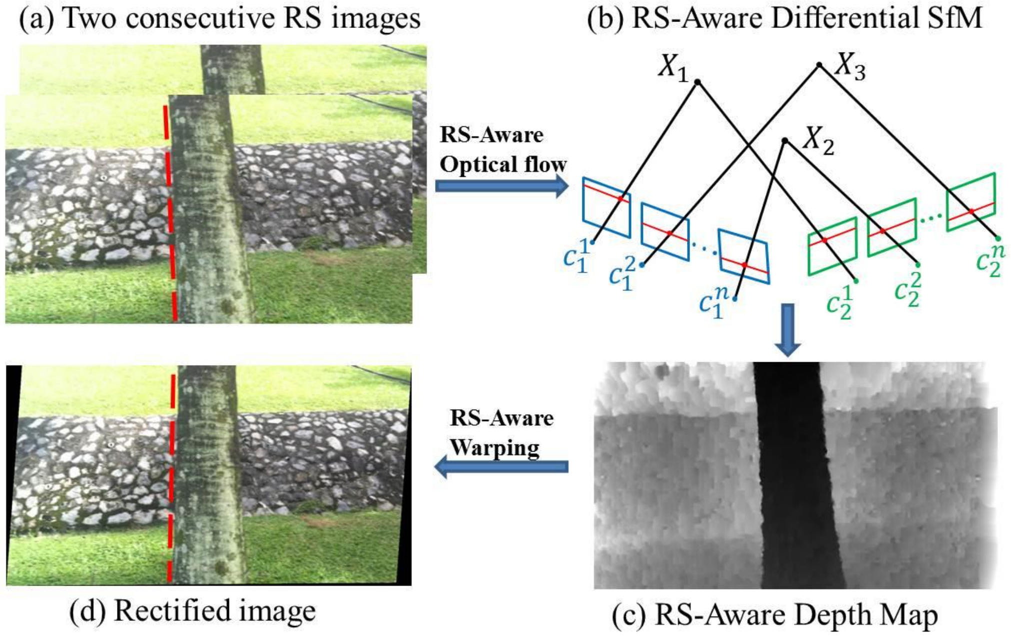
$$\mathbf{B}_i = \begin{bmatrix} \frac{x_i y_i}{f} & -\left(f + \frac{x_i^2}{f}\right) & y_i \\ \left(f + \frac{y_i^2}{f}\right) & -\frac{x_i y_i}{f} & -x_i \end{bmatrix},$$

with f being the camera focal length.

Zhuang B, Cheong L F, Hee Lee G. Rolling-shutter-aware differential sfm and image rectification. ICCV 2017.

Rolling Shutter Geometric Modeling and Optimization

Typical application 1: Differential motion model



RS-aware differential SfM and image rectification

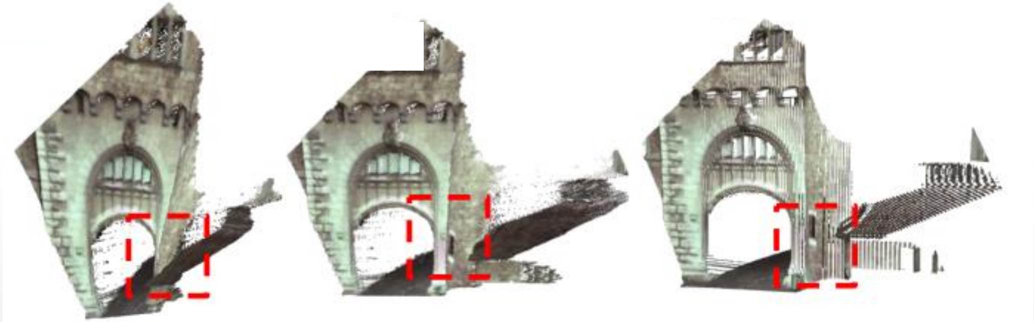
The **RS-aware differential re-projection geometry** between 3D scene flow and 2D optical flow can be modeled by a linear scaling operation: $\beta = 1 + \frac{\gamma \mathbf{f}_v}{h}$

$$\mathbf{f} = \beta \left[\frac{1}{Z} \begin{pmatrix} -f & 0 & x_i \\ 0 & -f & y_i \end{pmatrix} \boldsymbol{v} + \begin{pmatrix} \frac{x_i y_i}{f} & -\left(f + \frac{x_i^2}{f}\right) & y_i \\ \left(f + \frac{y_i^2}{f}\right) & -\frac{x_i y_i}{f} & -x_i \end{pmatrix} \boldsymbol{\omega} \right]$$

By further eliminating the RS depth Z , the **RS-aware differential epipolar constraint** under the constant velocity model is:

$$\frac{\mathbf{f}^T}{\beta} \hat{\boldsymbol{v}} \boldsymbol{x} - \boldsymbol{x}^T \boldsymbol{s} \boldsymbol{x} = 0$$

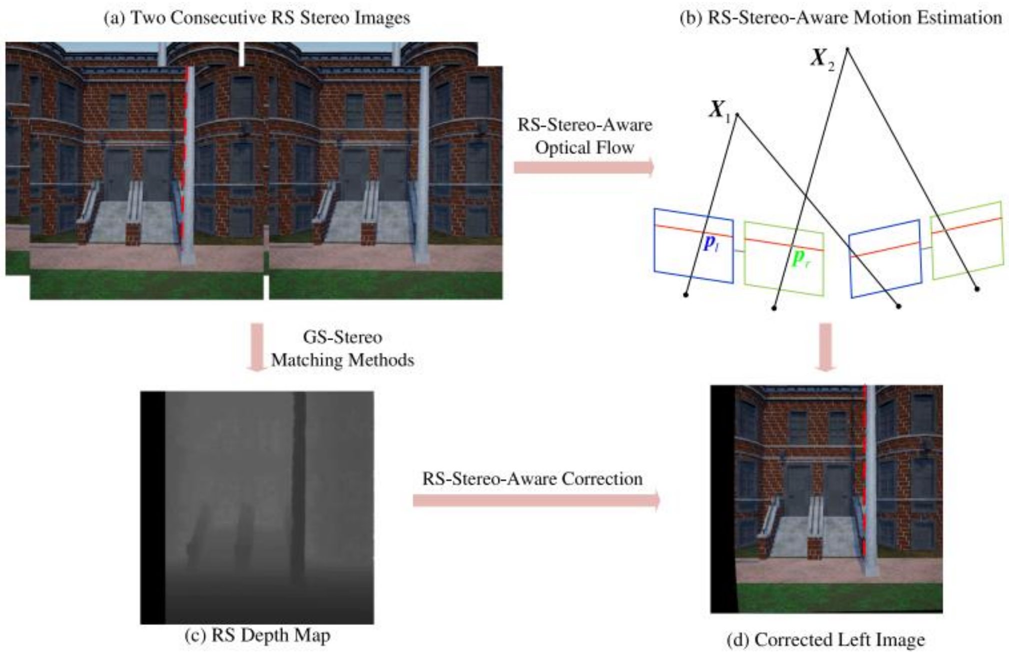
where $\boldsymbol{s} = \frac{1}{2}(\hat{\boldsymbol{v}}\hat{\boldsymbol{w}} + \hat{\boldsymbol{w}}\hat{\boldsymbol{v}})$. We can solve for the rolling shutter relative motion using conventional linear 8-point algorithm (Ma 2000).



Zhuang B, Cheong L F, Hee Lee G. Rolling-shutter-aware differential sfm and image rectification. ICCV 2017.
Ma Y, Košecká J, Sastry S. Linear differential algorithm for motion recovery: a geometric approach. IJCV 2000.

Rolling Shutter Geometric Modeling and Optimization

Typical application 2: Differential motion model



RS-stereo-aware differential SfM and image rectification

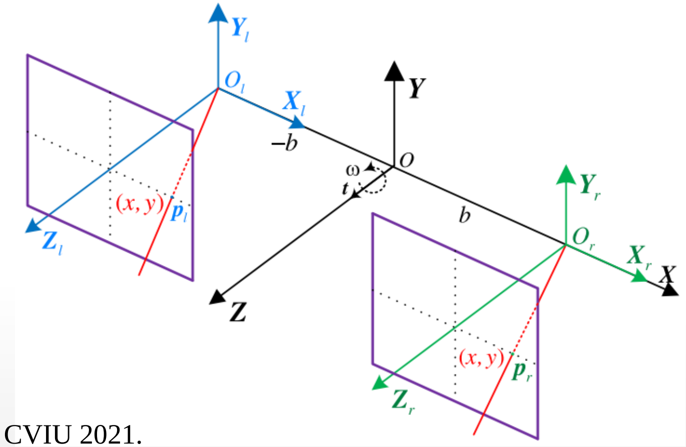
The **RS-stereo-aware differential re-projection geometry** between 3D scene flow and 2D optical flow can be modeled by a linear scaling operation: $\rho_i = 1 + \frac{\varphi v_i}{h}$

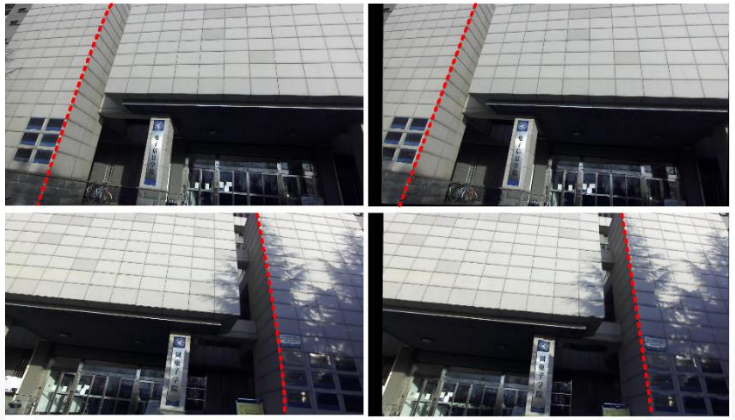
$$u_i = \rho_i \left[\frac{x(t_3 - b_i\beta) - f_x t_1}{Z_i} + \frac{\alpha xy}{f_y} - \beta\left(f_x + \frac{x^2}{f_x}\right) + \frac{\gamma f_x y}{f_y} \right] \triangleq \rho_i \left(\frac{1}{Z_i} u_i^{tr} + u_i^{rot} \right),$$

$$v_i = \rho_i \left[\frac{y(t_3 - b_i\beta) - f_y(t_2 + b_i\gamma)}{Z_i} - \frac{\beta xy}{f_x} + \alpha\left(f_y + \frac{y^2}{f_y}\right) - \frac{\gamma f_y x}{f_x} \right] \triangleq \rho_i \left(\frac{1}{Z_i} v_i^{tr} + v_i^{rot} \right).$$

By further eliminating the RS depth Z_i, the **RS-stereo-aware differential epipolar constraint** under the constant velocity model is:

$$\frac{1}{\rho_i}\left(u_i v_i^{tr} - v_i u_i^{tr}\right) = u_i^{rot} v_i^{tr} - v_i^{rot} u_i^{tr}, \quad i = l, r.$$



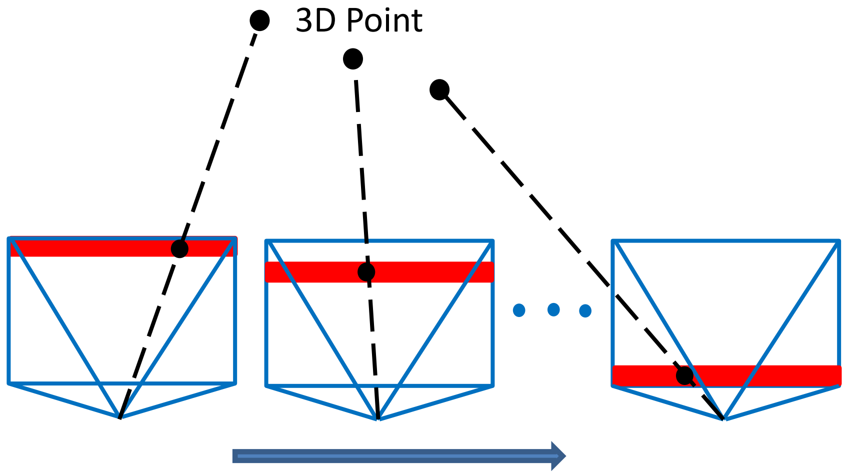


(a) Original RS left images (b) Corrected images (constant velocity)

Fan B, Dai Y, Wang K. Rolling-shutter-stereo-aware motion estimation and image correction. CVIU 2021.

Rolling Shutter Geometric Modeling and Optimization

Typical application 1: Uniform motion model



RS Perspective-n-point (**RnP**) problem

Single linearized model:

$$\alpha_i \begin{bmatrix} r_i \\ c_i \\ 1+\lambda(r_i^2+c_i^2) \end{bmatrix} = \mathtt{K}\left[(\mathtt{I}+(r_i-r_0)[\mathtt{w}]_\times)\,\mathtt{R}_0 \mid \mathtt{C}_0+(r_i-r_0)\mathtt{t}\right]\mathtt{X}_i$$

This model is rather complex. For calibrated RS camera and assuming Cayley parametrization of R_0, this model results in six equations of degree three in six unknowns and 64 solutions.

Double linearized model:

$$\alpha_i \begin{bmatrix} r_i \\ c_i \\ 1+\lambda(r_i^2+c_i^2) \end{bmatrix} = \mathtt{K}\left[(\mathtt{I}+(r_i-r_0)[\mathtt{w}]_\times)\,(\mathtt{I}+[\mathtt{v}]_\times) \mid \mathtt{C}_0+(r_i-r_0)\mathtt{t}\right]\mathtt{X}_i$$

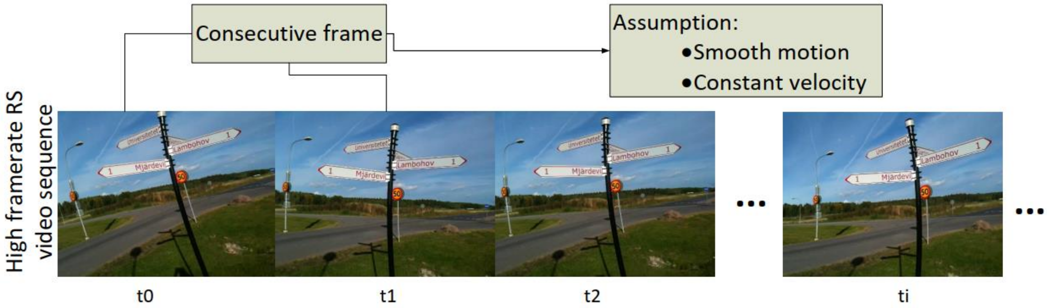
This model leads to a simpler way of solving the calibrated RS absolute pose from **≥ six 3D-2D point correspondences**.

Related Papers

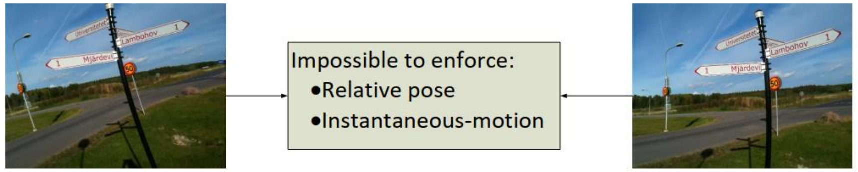
1. Albl C, Kukelova Z, Pajdla T. R6p-rolling shutter absolute camera pose. CVPR 2015.
2. Albl C, Kukelova Z, Pajdla T. Rolling shutter absolute pose problem with known vertical direction. CVPR 2016.
3. Kukelova Z, Albl C, Sugimoto A, et al. Linear solution to the minimal absolute pose rolling shutter problem. ACCV 2018.
4. Albl C, Kukelova Z, Larsson V, et al. Rolling shutter camera absolute pose. IEEE TPAMI 2019.
5. Kukelova Z, Albl C, Sugimoto A, et al. Minimal rolling shutter absolute pose with unknown focal length and radial distortion. ECCV 2020.
6. Albl C, Kukelova Z, Larsson V, et al. From two rolling shutters to one global shutter. CVPR 2020.

Rolling Shutter Geometric Modeling and Optimization

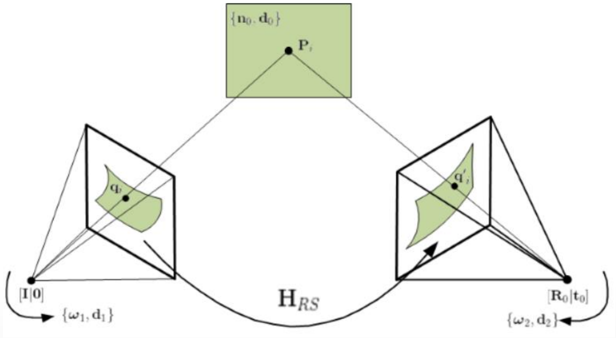
Typical application 2: Uniform motion model



(a) Frame-by-frame processing



(b) General unordered image set



General RS Homography

Rolling shutter Homography: $\alpha_i \mathbf{q}'_i = \mathbf{H}_{RS,i}\mathbf{q}_i$

$$
\begin{aligned}
\mathbf{H}_{RS,i} &= \mathbf{R}_i - \frac{\mathbf{t}_i \mathbf{n}_i^\top}{d_i} \\
&= (\mathbf{R}_0 + \mathbf{R}_1 v_i + \mathbf{R}_2 v'_i + \mathbf{R}_3 v_i v'_i) \\
&\quad + (\mathbf{t}_0 + \mathbf{t}_1 v_i + \mathbf{t}_2 v'_i + \mathbf{t}_3 v_i^2 + \mathbf{t}_4 v_i v'_i + \mathbf{t}_5 v_i^2 v'_i) \\
&\quad (\mathbf{N}_0 + \mathbf{N}_1 v_i) \\
&= \underbrace{(\mathbf{R}_0 + \mathbf{t}_0\mathbf{N}_0)}_{\mathbf{H}_{GS}} + \underbrace{(\mathbf{R}_1 + \mathbf{t}_1\mathbf{N}_0 + \mathbf{t}_0\mathbf{N}_1)}_{\mathbf{H}_1} v_i \\
&\quad + \underbrace{(\mathbf{R}_2 + \mathbf{t}_2\mathbf{N}_0)}_{\mathbf{H}_2} v'_i + \underbrace{(\mathbf{R}_3 + \mathbf{t}_4\mathbf{N}_0 + \mathbf{t}_2\mathbf{N}_1)}_{\mathbf{H}_3} v_i v'_i \\
&\quad + \underbrace{(\mathbf{t}_3\mathbf{N}_0 + \mathbf{t}_1\mathbf{N}_1)}_{\mathbf{H}_4} v_i^2 + \underbrace{(\mathbf{t}_5\mathbf{N}_0 + \mathbf{t}_4\mathbf{N}_1)}_{\mathbf{H}_5} v_i^2 v'_i \\
&\quad + \underbrace{(\mathbf{t}_3\mathbf{N}_1)}_{\mathbf{H}_6} v_i^3 + \underbrace{(\mathbf{t}_5\mathbf{N}_1)}_{\mathbf{H}_7} v_i^3 v'_i
\end{aligned}
$$

$$
\mathbf{H}_{RS,i} = \mathbf{H}_{GS} + \mathbf{H}_1 v_i + \mathbf{H}_2 v'_i + \mathbf{H}_3 v_i v'_i + \mathbf{H}_4 v_i^2 + \mathbf{H}_5 v_i^2 v'_i + \mathbf{H}_6 v_i^3 + \mathbf{H}_7 v_i^3 v'_i
$$

To estimate the full rolling shutter homography matrix, **at least 36 2D-2D point correspondences** are required. A DLT solution can be obtained by using SVD.

Lao Y, Ait-Aider O. Rolling shutter homography and its applications. IEEE TPAMI 2020.

Typical application 3: Uniform motion model

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Yuchao Dai, Hongdong Li, Laurent Kneip

CVPR 2016

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Rolling Shutter Models:

- A rolling shutter camera does no longer possess a single center-of-projection in the general case. Instead, each of its scanlines generally has a different projection center (**temporal-dynamic**) as well as a different local frame and orientation.

When an RS camera is in motion during image acquisition, all its scanlines are sequentially exposed at different time steps; hence each scanline possesses a different local frame. Mathematically, we need to assign a unique projection matrix to every scanline in an RS image. For example, for the u_i-th scanline, we have

$$\mathbf{P}_{u_i} = \mathbf{K}[\mathbf{R}_{u_i}, \mathbf{t}_{u_i}].$$

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Rolling Shutter Models:

- Linear rolling shutter camera:

$$\mathbf{P}_{u_i} = [\mathbf{R}_0, \mathbf{t}_0 + u_i\mathbf{d}].$$

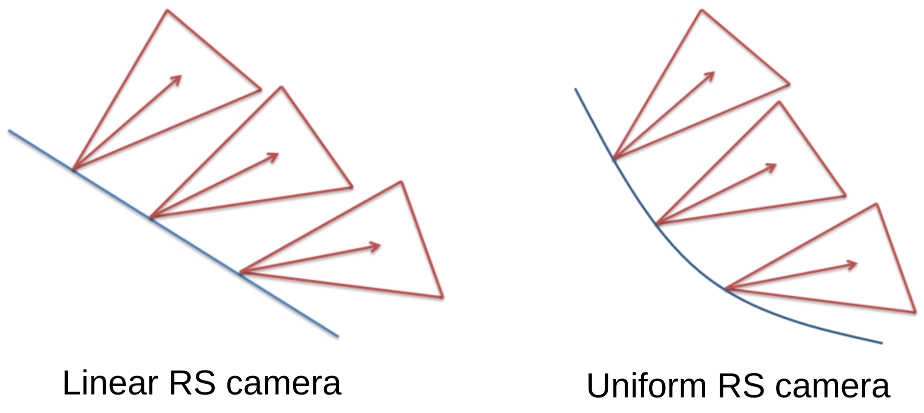
We use the top-most scanline's local frame $[\mathbf{R}_0, \mathbf{t}_0]$ as the reference frame of the RS image

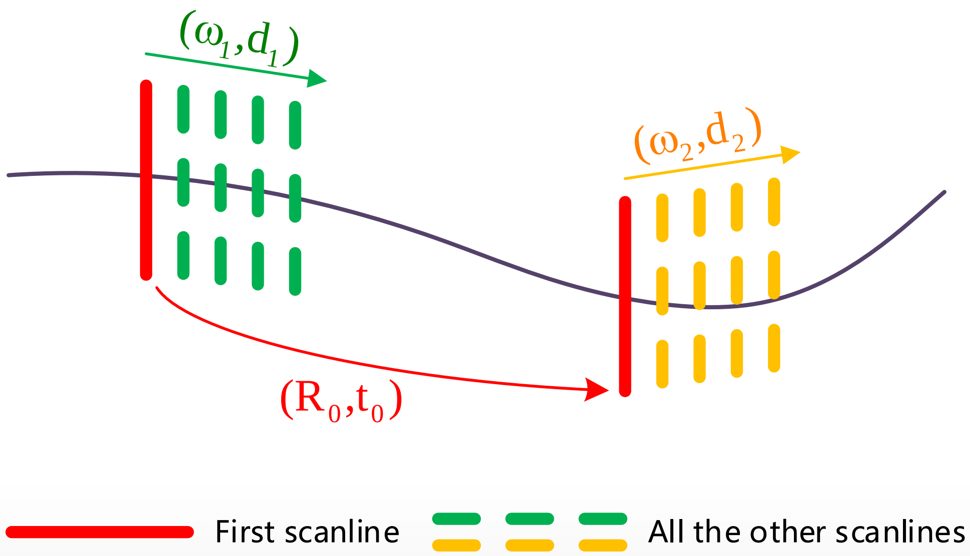
- Uniform rolling shutter camera:

$$\mathbf{R}_{u_i} = (\mathbf{I} + \sin(u_i\omega)[\mathbf{n}]_\times + (1 - \cos(u_i\omega))[\mathbf{n}]_\times^2)\mathbf{R}_0,$$
$$\mathbf{t}_{u_i} = \mathbf{t}_0 + u_i\mathbf{d}.$$

Under the small-rotation approximation, we have

$$\mathbf{R}_{u_i} = (\mathbf{I} + u_i\omega[\mathbf{n}]_\times)\mathbf{R}_0,$$
$$\mathbf{t}_{u_i} = \mathbf{t}_0 + u_i\mathbf{d}.$$





Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

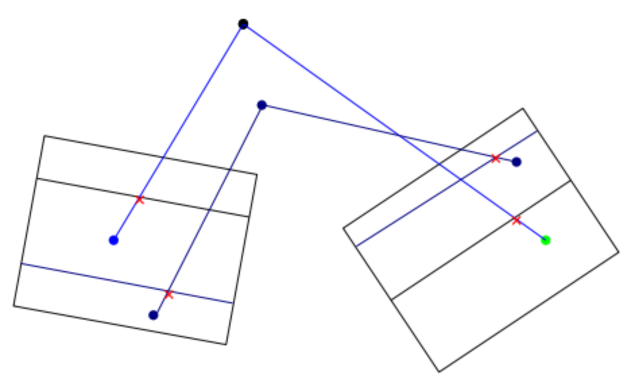
Rolling Shutter Relative Pose:

For a rolling-shutter camera, every scanline has its own distinct local pose. As a result, every pair of feature correspondences may give rise to a different "essential matrix". Formally, for $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$, we have

$$\mathbf{x}'^{T}_i \mathbf{E}_{u_i, u'_i} \mathbf{x}_i = 0.$$

Note that **E** is dependent of the scanlines u_i and u'_i . In other words, there does not exist a single global 3×3 essential matrix for a pair of RS images. Given two scanlines u_i, u_j and the corresponding camera poses $\mathbf{P}_{u_i} = [\mathbf{R}_{u_i}, \mathbf{t}_{u_i}]$ and $\mathbf{P}_{u_j} = [\mathbf{R}_{u_j}, \mathbf{t}_{u_j}]$, we have

$$\mathbf{E}_{u_i u_j} = [\mathbf{t}_{u_j} - \mathbf{R}_{u_j}\mathbf{R}^T_{u_i}\mathbf{t}_{u_i}]_\times \mathbf{R}_{u_j}\mathbf{R}^T_{u_i}.$$



Despite the fact that different scanlines possess different centers of projection, for a pair of feature correspondences the co-planarity relationship still holds. As such, the concept of two-view epipolar relationship should still exist.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Rolling Shutter Essential Matrices:

- Generalize the conventional 3×3 essential matrix for perspective cameras
- Derive 5×5 and 7×7 essential matrices for different types of Rolling-Shutter (RS) cameras
- Filling the gap of 4×4 and 6×6 essential matrices for Push-Broom (PB) cameras

Table 1. A hierarchy of generalized essential matrices for different types of rolling-shutter and push-broom cameras.

Camera Model	Essential Matrix	Monomials	Degree-of-freedom	Linear Algorithm	Non-linear Algorithm	Motion Parameters
Perspective camera	$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$	$(u_i, v_i, 1)$	$3^2 = 9$	8-point	5-point	$\mathbf{R}, \mathbf{t}$
Linear push broom	$\begin{bmatrix} 0 & 0 & f_{13} & f_{14} \\ 0 & 0 & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix}$	$(u_i v_i, u_i, v_i, 1)$	$12 = 4^2 - 2^2$	11-point	11-point	$\mathbf{R}, \mathbf{t}, \mathbf{d}_1, \mathbf{d}_2$
Linear rolling shutter	$\begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} \\ 0 & 0 & f_{23} & f_{24} & f_{25} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} \end{bmatrix}$	$(u_i^2, u_i v_i, u_i, v_i, 1)$	$21 = 5^2 - 2^2$	20-point	11-point	$\mathbf{R}, \mathbf{t}, \mathbf{d}_1, \mathbf{d}_2$
Uniform push broom	$\begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} & f_{16} \\ 0 & 0 & f_{23} & f_{24} & f_{25} & f_{26} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} \end{bmatrix}$	$(u_i^2 v_i, u_i^2, u_i v_i, u_i, v_i, 1)$	$32 = 6^2 - 2^2$	31-point	17-point	$\mathbf{R}, \mathbf{t}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{d}_1, \mathbf{d}_2$
Uniform rolling shutter	$\begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} & f_{16} & f_{17} \\ 0 & 0 & f_{23} & f_{24} & f_{25} & f_{26} & f_{27} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} & f_{37} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} & f_{47} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} & f_{57} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} & f_{67} \\ f_{71} & f_{72} & f_{73} & f_{74} & f_{75} & f_{76} & f_{77} \end{bmatrix}$	$(u_i^3, u_i^2 v_i, u_i^2, u_i v_i, u_i, v_i, 1)$	$45 = 7^2 - 2^2$	44-point	17-point	$\mathbf{R}, \mathbf{t}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{d}_1, \mathbf{d}_2$

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Example 1: A 5×5 essential matrix for linear RS cameras

For a linear rolling shutter camera, since the inter-scanline motion is a pure translation, there are four parameter vectors to be estimated, namely $\{\mathbf{R}, \mathbf{t}, \mathbf{d}_1, \mathbf{d}_2\}$. The total degree of freedom of the unknowns is $3+3+3+3-1=11$.
The epipolarity defined between the u_i-th scanline of the first RS frame and the u_i'-th scanline of the second RS frame is represented as $\mathbf{E}_{u_i u_i'} = [\mathbf{t}_{u_i u_i'}]_\times \mathbf{R}_{u_i u_i'}$, where the translation $\mathbf{t}_{u_i u_i'} = \mathbf{t} + u_i'\mathbf{d}_2 - u_i\mathbf{R}\mathbf{d}_1$. This translates into

$$\begin{bmatrix} u_i' \\ v_i' \\ 1 \end{bmatrix}^T [\mathbf{t} + u_i'\mathbf{d}_2 - u_i\mathbf{R}\mathbf{d}_1]_\times \mathbf{R} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = 0. \quad (7)$$

Expanding this scanline epipolar equation, one can obtain the following 5×5 matrix form:

$$\begin{bmatrix} u_i'^2 \\ u_i' v_i' \\ u_i' \\ v_i' \\ 1 \end{bmatrix}^T \begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} \\ 0 & 0 & f_{23} & f_{24} & f_{25} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} \end{bmatrix} \begin{bmatrix} u_i^2 \\ u_i v_i \\ u_i \\ v_i \\ 1 \end{bmatrix} = 0, \quad (8)$$

where the entries of the 5×5 matrix $\mathbf{F} = [f_{i,j}]$ are functions of the 11 unknown parameters $\{\mathbf{R}, \mathbf{t}, \mathbf{d}_1, \mathbf{d}_2\}$. In total, there are 21 homogeneous variables, thus a linear 20-point solver must exist to solve for this hyperbolic essential matrix.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Example 1: A 5×5 essential matrix for linear RS cameras

- Proof:

By redefining $\mathbf{d}_1 \leftarrow \mathbf{R}\mathbf{d}_1$, we easily obtain

$$\mathbf{E}_{u_i u_i'} \quad = \quad ([\mathbf{t}]_\times + u_i'[\mathbf{d}_2]_\times - u_i[\mathbf{d}_1]_\times)\,\mathbf{R}. \tag{9}$$

Denoting $\mathbf{E}_0 = [\mathbf{t}]_\times \mathbf{R}$, $\mathbf{E}_1 = [\mathbf{d}_1]_\times \mathbf{R}$ and $\mathbf{E}_2 = [\mathbf{d}_2]_\times \mathbf{R}$, we have:

$$[u_i', v_i', 1](\mathbf{E}_0 + u_i'\mathbf{E}_2 - u_i\mathbf{E}_1)[u_i, v_i, 1]^T = 0. \tag{10}$$

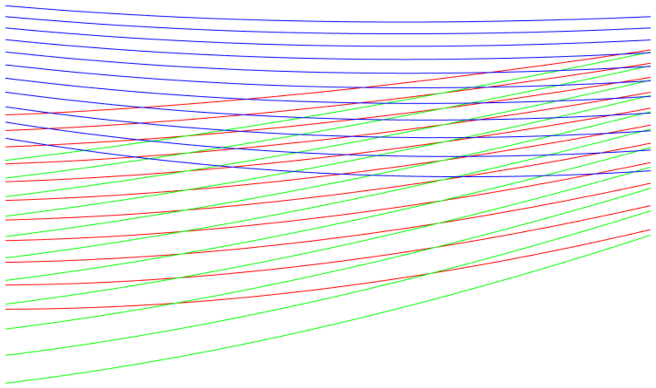
The 5×5 matrix $\mathbf{F}$ is defined in the following way

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & E_{1,11} & E_{1,21} & E_{1,31} \\ 0 & 0 & E_{1,12} & E_{1,22} & E_{1,32} \\ E_{2,11} & E_{2,21} & a & b & c \\ E_{2,12} & E_{2,22} & E_{0,12} + E_{2,32} & E_{0,22} & E_{0,32} \\ E_{2,13} & E_{2,23} & E_{0,13} + E_{2,33} & E_{0,23} & E_{0,33} \end{bmatrix}, \tag{11}$$

where $a = E_{0,11} + E_{1,13} + E_{2,31}$, $b = E_{0,21} + E_{1,23}$, $c = E_{0,31} + E_{1,33}$. Finally, it is easy to verify the equation

$$[u_i'^2, u_i'v_i', u_i', v_i', 1]\mathbf{F}[u_i^2, u_iv_i, u_i, v_i, 1]^T = 0.$$

The "epipolar lines" for a linear RS camera are hyperbolic curves. It is easy to verify that the generalized essential matrix for linear rolling shutter camera is full rank and the epipole lies in infinity.



"epipolar lines"

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

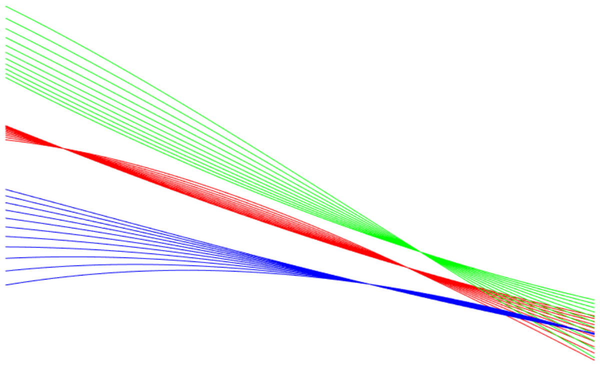
Example 2: A 7×7 essential matrix for uniform RS cameras

Consider a uniform RS camera undergoing a rotation at constant angular velocity $\mathbf{w}$ and a translation at constant linear velocity $\mathbf{d}$. We assume the angular velocity is very small. By using the small-rotation approximation,we have the u_i-th scanline's local pose as

$$\mathbf{P}_{u_i} = [(\mathbf{I} + u_i[\mathbf{w}]_\times)\mathbf{R}_0,\ \mathbf{t}_0 + u_i\mathbf{d}]. \tag{12}$$

Given a pair of two corresponding uniform RS camera frames, we then have

$$[u_i', v_i', 1][\mathbf{t} + u_i'\mathbf{d}_2 - u_i\mathbf{R}_{u_iu_i'}\mathbf{d}_1]_\times \mathbf{R}_{u_iu_i'}[u_i, v_i, 1]^T = 0, \tag{13}$$



“epipolar lines”

Expanding this equation with the aid of the small rotation approximation results in

$$\mathbf{R}_{u_i,u_i'} = (\mathbf{I} + u_i'[\mathbf{w}_2]_\times)\mathbf{R}_0(\mathbf{I} - u_i[\mathbf{w}_1]_\times), \tag{14}$$

and we finally obtain:

$$\left[u_i'^3, u_i'^2v_i', u_i'^2, u_i'v_i', u_i', v_i', 1\right] \mathbf{F} \left[u_i^3, u_i^2v_i, u_i^2, u_iv_i, u_i, v_i, 1\right]^T = 0, \tag{15}$$

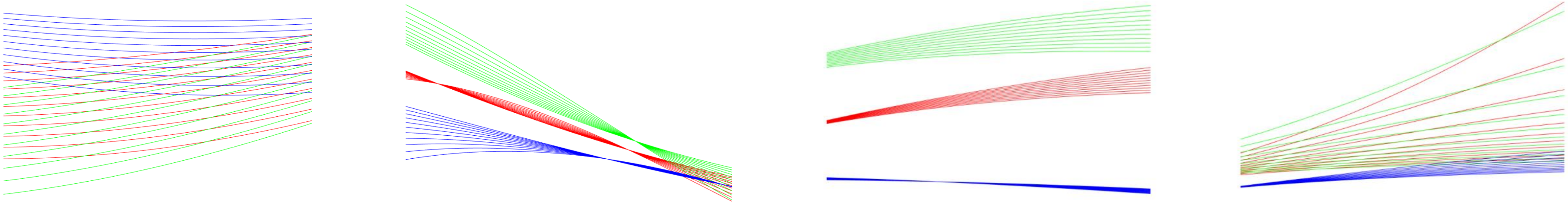
where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} & f_{16} & f_{17} \\ 0 & 0 & f_{23} & f_{24} & f_{25} & f_{26} & f_{27} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} & f_{37} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} & f_{47} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} & f_{57} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} & f_{67} \\ f_{71} & f_{72} & f_{73} & f_{74} & f_{75} & f_{76} & f_{77} \end{bmatrix}.$$

This gives a 7×7 RS essential matrix $\mathbf{F}$, whose elements are functions of the 18 unknowns (i.e. $\{\mathbf{R}, \mathbf{t}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{d}_1, \mathbf{d}_2\}$). Also note the induced epipolar curves are *cubic*.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Epipolar Curves



Linear RS　　Uniform RS　　Linear PB　　Uniform PB

RS camera vs PB camera

Both RS camera and PB camera have a scanline dependent pose, i.e., temporal-dynamic center of projection. For PB cameras, the scanline direction is fixed relative to the local coordinate while the scanline direction changes with respect to the local coordinate for RS cameras. This creates the main difference between PB cameras and RS cameras and the extras freedom explains the increased order of polynomials in expressing the generalized epipolar geometry (4 VS 6 and 5 VS 7).

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Linear N-point algorithms for RS cameras:

- Let us use as an example the **linear RS camera** to derive a **linear 20-point algorithm** for solving the linear RS essential matrix. The linear solutions for other types of cameras in the table can be similarly derived.

◆ (1) Solving the 5×5 linear RS essential matrix:

The linear RS essential matrix $\mathbf{F}$ contains only 21 non-trivial homogeneous variables, hence its degree of freedom is 20. Collecting 20 correspondences, one can solve for the 5×5 matrix $\mathbf{F}$ linearly by SVD.

◆ (2) Recovering atomic essential matrices:

Once the 5×5 matrix $\mathbf{F}$ is found, our next goal is to recover the individual atomic essential matrices $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$. Eq.-(11) provides 21 linear equations on the three essential matrices. As the three essential matrices consist of 27 elements, we need six extra constraints to solve for $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$. To this end, we resort to the inherent constraints on standard 3×3 essential matrices, $\det(\mathbf{E}) = 0$ and $2\mathbf{E}\mathbf{E}^T\mathbf{E} - \mathbf{Tr}(\mathbf{E}\mathbf{E}^T)\mathbf{E} = 0$, since $\mathbf{E}_0, \mathbf{E}_1$ and $\mathbf{E}_2$ are standard 3×3 essential matrices.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Nonlinear geometric errors for RS cameras:

- Normalization:

In solving the linear RS essential matrix $\mathbf{F}$, it is important to implement a proper normalization: 1) Normalizing the image coordinates data (u_i, v_i) and $(u_i^{'}, v_i^{'})$ in the way as described in [Hartley 1997]. 2) Under the linear rolling shutter relative pose formulation, the inputs are monomials $(u_i^2, u_i v_i, u_i, v_i, 1)$ and $(u_i^{'2}, u_i^{'} v_i^{'}, u_i^{'}, v_i^{'}, 1)$, a better normalization should be defined on $(u_i^2, u_i v_i, u_i, v_i, 1)$ and $(u_i^{'2}, u_i^{'} v_i^{'}, u_i^{'}, v_i^{'}, 1)$ rather than (u_i, v_i) and $(u_i^{'}, v_i^{'})$. Therefore, we propose to normalize $(u_i^2, u_i v_i, u_i, v_i, 1)$ and $(u_i^{'2}, u_i^{'} v_i^{'}, u_i^{'}, v_i^{'}, 1)$ in the way as in [Hartley 1997].

- Nonlinear Solvers w/ Sampson Error:

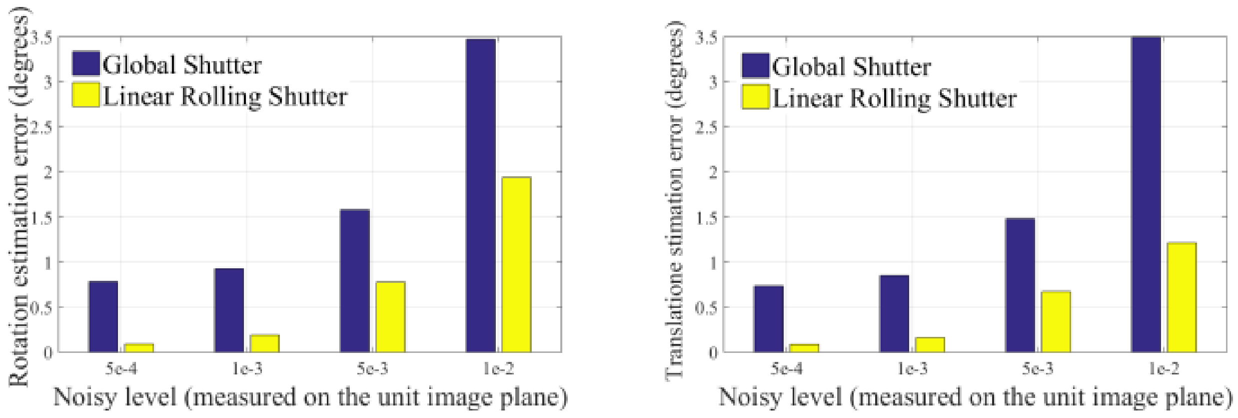
Based on the above generalized essential matrices, we can now also devise nonlinear solvers. Instead of minimizing an algebraic error, we minimize the geometrically more meaningful (generalized) Sampson error metric. For example, in the case of a uniform RS camera, the Sampson error is the first-order approximation of the distance between a (generalized) feature vector $\mathbf{x}_i = [u_i^3, u_i^2 v_i, u_i^2, u_i v_i, u_i, v_i, 1]^T$ and its corresponding RS epipolar curve, i.e.,

$$e_{\text{Sampson}} = \sum_{i=1}^{n} \frac{(\mathbf{x}_i^{'T} \mathbf{F} \mathbf{x}_i)^2}{\sum_{j=1}^{7}((\mathbf{F}\mathbf{x}_i)_j^2 + (\mathbf{F}^T \mathbf{x}_i^{'})_j^2)}. \tag{20}$$

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Experiments

- Accuracy versus noise level



(a) Rotation estimation error (b) Translation estimation error

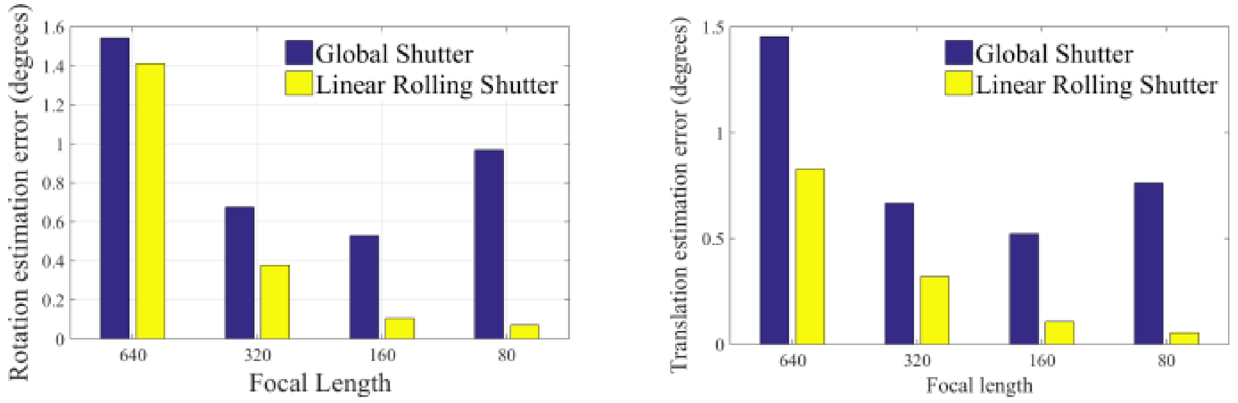
Figure: Performance evaluation with increasing Gaussian noise.

Our linear RS camera model always achieves better performance than the global shutter camera model, while both rotation and translation errors increase with increasing noise level.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Experiments

- Accuracy versus focal-length



(a) Rotation estimation error (b) Translation estimation error

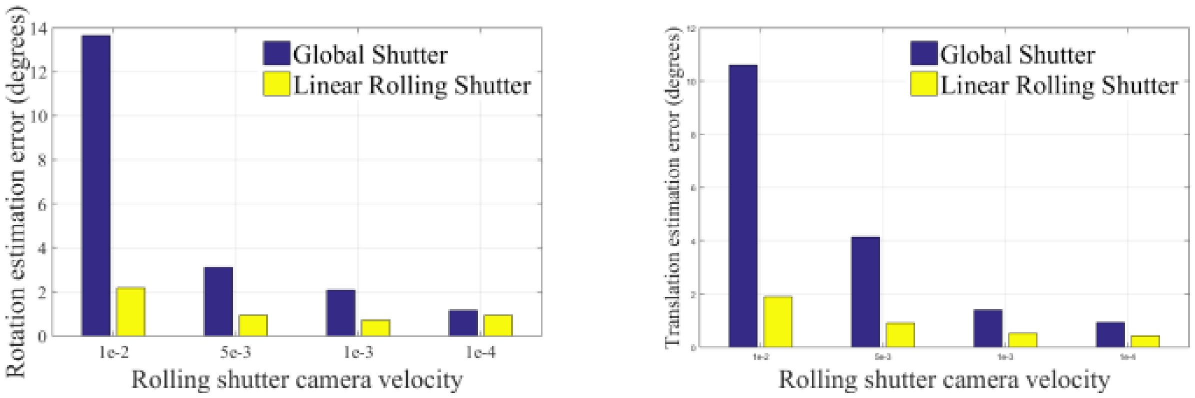
Figure: Evaluation on decreasing focal length.

With a decreasing focal length, the RS effect becomes increasingly well observable, leading to a decrease of the motion estimation error.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

Experiments

- Accuracy versus RS velocity



(a) Rotation estimation error (b) Translation estimation error

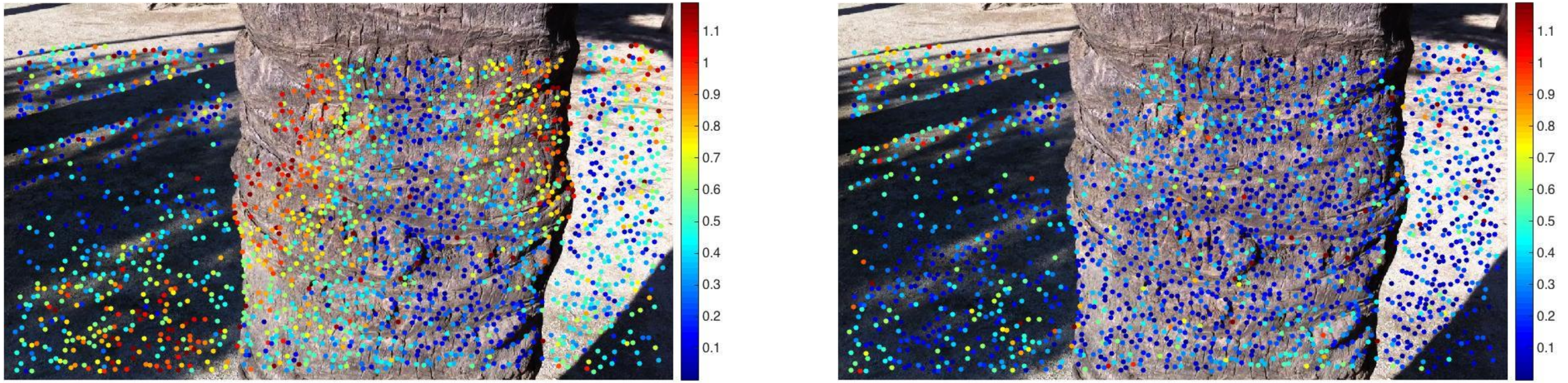
Figure: Evaluation over decreasing translation velocity.

With an increasing velocity, our linear RS model achieves an obvious improvement in pose estimation, which suggests that the RS effect is more observable under large linear and angular motion.

Rolling Shutter Camera Relative Pose: Generalized Epipolar Geometry

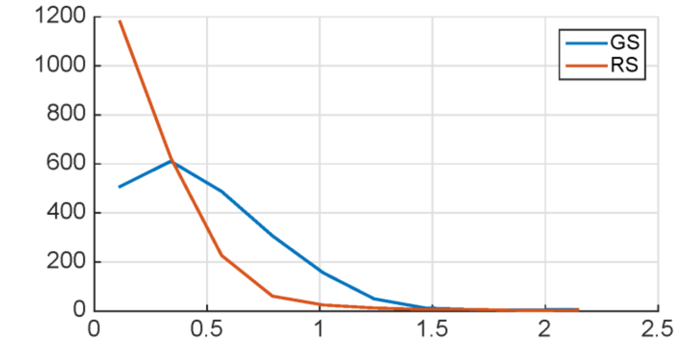
Experiments

- Test on real RS images



(a) Global shutter model (b) Rolling shutter model

Comparisons of the Sampson errors for a pair of images taken from a RS video dataset. (a) shows the final result of Sampson error minimization based on a global shutter model. The error distribution has a structure in the image plane, indicating regions for which the RS distortion is not properly taken into account. (b) shows how the inclusion of a RS model and the extended Sampson distance take those distortions into account, and produce a reprojection error that distributes much more uniformly across the entire image plane.



Error Histogram

Conclusions

- Novel generalized essential matrices of size 4×4, 5×5, 6×6, and 7×7 for linear PB, linear RS, uniform PB, and uniform RS cameras, respectively.
- Effective linear N-point algorithms and non-linear Sampson error minimizers for solving these generalized essential matrices.
- The entire work represents a unified and elegant framework for solving the Relative Pose problem with new types of cameras, including the practically relevant and previously unsolved case of a RS camera Potential extensions : light-field cameras, general linear cameras, and generalized camera models.
- The theory promises a more general applicability to spatio-temporally scanning sensors, such as satellite imagery and sweeping Laser scanners.

For more content, see our recent rolling shutter review paper:

Bin Fan, Yuchao Dai*, Mingyi He. Rolling Shutter Camera: Modeling, Optimization and Learning[J]. Machine Intelligence Research, 2022.

THANK YOU FOR WATCHING